DESIGN OF A TENSION ROOF STRUCTURE
FOR THE
ROCKWELL CAGE

BY

DANIEL W. EGGERS

BACHELOR OF SCIENCE IN STRUCTURAL ENGINEERING/ARCHITECTURE
JUNE 1997
Princeton University

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING IN CIVIL AND ENVIRONMENTAL ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2000

SIGNATURE OF AUTHOR ______________________________
DANIEL W. EGGERS
MAY 15, 2000

CERTIFIED BY ______________________________
JEROME J. CONNOR
PROFESSOR, DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING
THESIS SUPERVISOR

APPROVED BY ______________________________
DANIELE VENEZIANO
CHAIRMAN, DEPARTMENTAL COMMITTEE ON GRADUATE STUDIES

DESIGN OF A TENSION ROOF STRUCTURE
FOR THE
ROCKWELL CAGE

BY

DANIEL W. EGGERS

Submitted to the Department of Civil and Environmental Engineering on
May 5, 2000
in Partial Fulfillment of the Requirements For the Degree of
Master of Engineering in Civil and Environmental Engineering

ABSTRACT

The Rockwell Cage presents an ideal opportunity for application of a cable net structure. This study examines tensile structures and means of designing and determining their behavior. Hand calculations are derived and used to determine a first run approximation of structural response. In search of more accurate results, commercially available non-linear software was used unsuccessfully. Therefore, an exact non-linear matrix analysis was used to solve the system and examine the parameters which affect the structure.

The analysis revealed that the hand calculations provide an adequate first-approximation, but are unreliable in terms of stiffness. The non-linear matrix model provides greater accuracy over a wide range of parameters. Furthermore, the non-linear matrix model provides significantly more flexibility in terms of various loading patterns, structural forms, and the ability to easily examine parameters. The working program is explained and included in this thesis.

THESIS SUPERVISOR: JEROME J. CONNOR

TITLE: PROFESSOR OF CIVIL AND ENVIRONMENTAL ENGINEERING

ACKNOWLEDGEMENTS

I would like to thank Professor Connor for all of his help and guidance throughout the year, and especially on this thesis. His patience and suggestions allowed this thesis to exist. I would also like to thank the entire MIT High Performance Structures Program of 2000. Their humor and wisdom made this past year enjoyable and enlightening.

To my family and friends: Thank you for your support this year and always. Thank you Erin for all the love, support, help, and patience you have shown during the writing of this thesis.

Table of Contents

TABLE OF FIGURES

1 Introduction

> "Experience has shown that, although in most cases the basic concept is simple, lack of acquaintance with cable structures and their behavior can lead to an unnecessary waste of time and consequently high design costs (Buchholdt)."

Cable net structures use the most efficient form of load transmission, tension, which avoids concerns of buckling. Furthermore these structures can be architecturally and structurally pleasing. Despite their advantages, tension structures are not frequently seen in the built environment. Their under-utilization is generally attributed to a lack of guidance in building codes, lack of information, and a shortage of trained designers and contractors familiar with this type of structure.

Cable net structures are difficult to design because they behave non-linearly. Furthermore, the stiffness and strength of these structures depends on numerous parameters including: cable type, cable size, pretension levels, and support stiffness. The purpose of this thesis is to examine cable net structures with an emphasis on design. The intent of this thesis is to increase the designer's awareness, acquaintance and knowledge of tensile structures. The Rockwell Cage at MIT serves as the design basis for examining these structures.

1.1 The Rockwell Cage

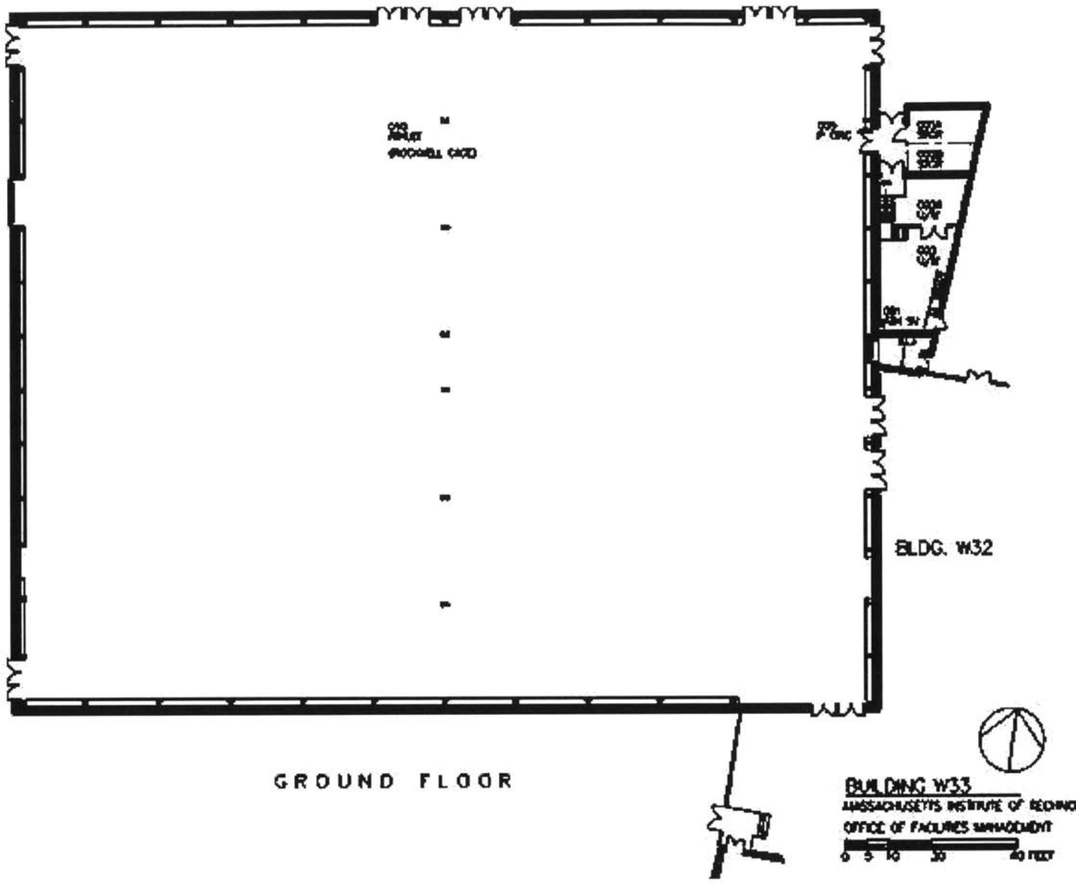


Figure 1: The Rockwell Cage, Plan View (MIT)

The Rockwell Cage at the Massachusetts Institute of Technology was formerly a hanger for lightweight aircraft. It was constructed in 1948 using post World War II military surplus timber for a double barrel vaulted roof supported by wood roof trusses. The structure was later modified to meet MIT's athletic needs. The building measures 196 feet by 156 feet (approximately 48 meters by 30 meters) with W18 steel columns 36' high spaced at 16'-4" on center. The walls are CMU from the ground level to 12' and the remainder is a clear glass window. The lateral load of the structure is resisted by steel single angle cross-bracing. The roof vaults span 98 feet dividing the longer dimension in half and creating a row of columns in the center of the building. The wood trusses rest on steel trusses supported by steel columns at the center and each end of the building. The space is used for basketball, volleyball and other court activities.

The Rockwell Cage is far from an ideal athletic facility. The half of the wall that is constructed of window panels is subsequently covered by thick curtains. This has been done to block the sun glare as well as to insulate the building. The floor of the Rockwell Cage is undergoing differential settlement at the walls which is damaging the floor surface. Finally, the row of the columns in the center of the building limits the flexibility of the space.

Alterations to the Rockwell Cage have been suggested in the past. MIT's Materials and Construction 4.405 studied possible steel and concrete architectural alternatives for the Rockwell Cage in the Fall of 1999. In fact, MIT plans to construct two new athletic facilities, the second of these involves the demolition and replacement of the Rockwell Cage. The new building will be placed on the site of the old and will contain basketball courts, racquetball courts and a multipurpose room (Blau 1996).

This thesis uses Rockwell Cage as a framework for studying the design of a tension net structure. The building function lends itself well to a long-span structure. MIT's campus is an ideal location for an unusual structural type, especially in the location of the Rockwell Cage, surrounded, as it is, by a concrete shell and a pneumatic dome.

2 INTRODUCTION TO CABLE STRUCTURES

Cable structures carry load in the most efficient form possible, tension. Tension members carry the load axially and do not require bracing for buckling. However, they can be costly to construct due to complicated erection and connection procedures. As a result, they are not cost effective structures for spans of less than 30 meters but are highly economical for spans over 100 meters (Buchholdt).

The simplest method of supporting a roof by means of a cable is to attach the roofing material to a series of cables (Figure 2). These cables are draped in a fashion capable of carrying the load. This system has no inherent stiffness and will move significantly under live loads. The greatest threat to the suspended roof is uplift forces. If the uplift force becomes large enough to equal the dead load of the roof, the roof will have no stiffness. Any increase in uplift forces would then cause a drastic deflection. This situation may be avoided by using a heavy cladding material such as concrete slabs to increase the dead load. Stiffness may also be added to the structure by using the concrete as a rigid shell-like membrane.

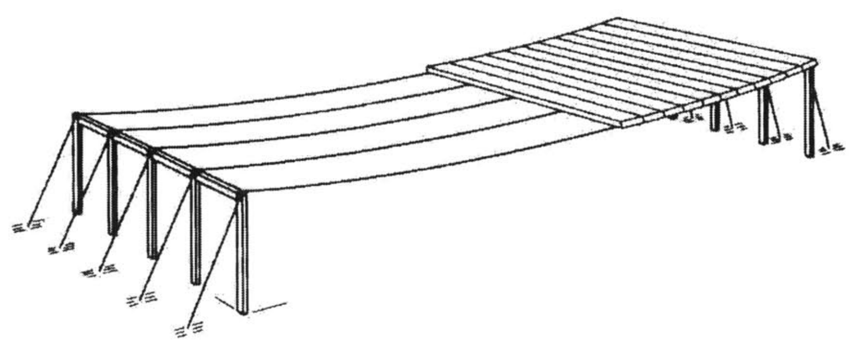
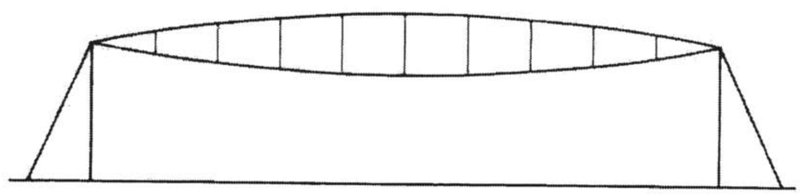
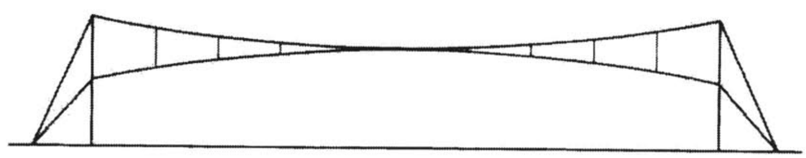


Figure 2: Suspended roof structure (Buchholdt pg. 3).

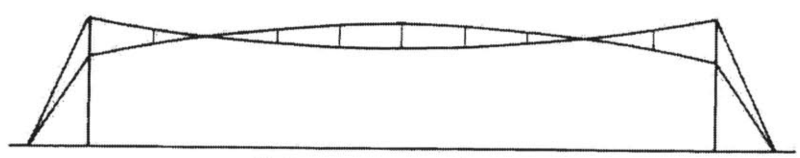
An alternate method of ensuring that the suspension cables remain in tension is to install a pretension cable having an opposite curvature to that of the suspension cable. This second cable pulls downwards on the suspension cable with a force sufficient to force the suspension cable into tension under all loading conditions. Loads placed on the structure cause an increased tension in the suspension cable and a decreased tension in the pretension cable. The opposite is true under uplift conditions. This arrangement is called a cable beam (Figure 3).



(a) Convex cable beam



(b) Concave cable beam



(c) Convex–concave cable beam

Figure 3: Cable beams (Buchholdt, pg. 5).

This system is stable as long as the load does not cause a cable to lose all tension. In the event of loss of tension in a cable, the structure is essentially the same as a single cable and will thus have a greatly decreased stiffness. Such a loss of stiffness may occur either globally or locally. A localized 'soft spot' can also exist where the cables are relatively flat, resulting in a low stiffness. A structure with the suspension and pretension cables having opposite, constant curvatures will avoid this phenomenon.

Slightly more complex than the cable beam is the cable net. A cable net is a system of cables which forms a single surface. The suspension and pretension cables are arranged perpendicularly in plan, forming a single three-dimensional surface. Cable nets can be made in various forms. To achieve the criteria of opposite, constant curvature as well as reasonable drainage, a hyperbolic paraboloid form is required (Figure 4). This same form can be used with various boundaries and cuts to create a number of different forms (Figure 5).

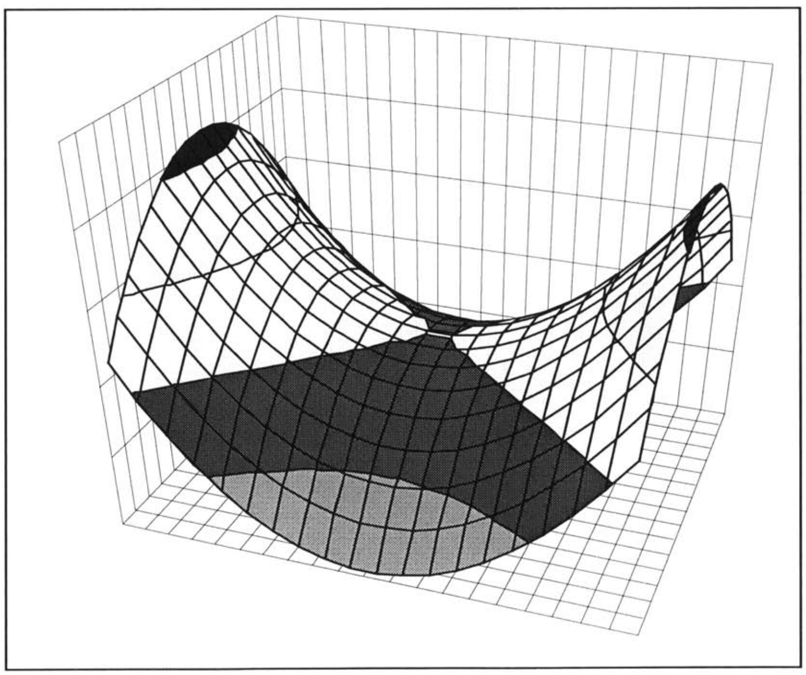


Figure 4: A hyperbolic paraboloid.

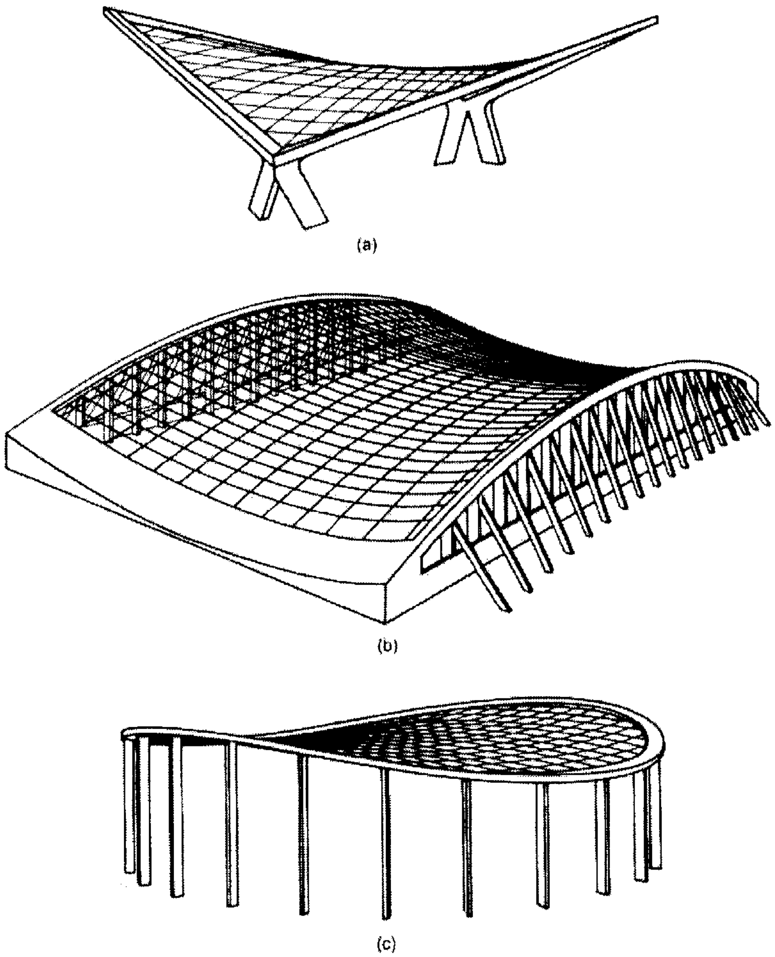


Figure 5: Three cable net structres (Buchholdt pg. 10).

An even greater variety of structural forms are available than are explained by the hyperbolic paraboloid. These can be achieved by the use of compression elements such as masts and arches. A circus tent is an example of such a form. Another example is the Hong Kong Aviary, which uses three compression arches to support a cable net (Figure 6).

Figure 6: The Hong Kong Aviary

For the calculations in section 4, the elliptical edge ring, similar to that shown in Figure 5c, was chosen. The primary advantage of the edge ring is the ability to resist the tension in the cables in the ring without ground anchors. The disadvantage may be that it would not fit a rectangular building plan. However such a ring could still be supported and the corners covered by other means.

2.1 CABLE BEHAVIOR

Cables have the unique property of being able to carry load in tension only. They do not allow for compression or shear loading. Therefore, a cable under a particular loading must assume a shape such that all of the load is carried in tension. The load which defines this initial shape of the cable is called the funicular load, and the shape is called the funicular curve. Changes in the magnitude of the funicular load do not alter the deformed shape, just the magnitude of the sag.

Application of a non-funicular load requires the cable to alter its deformed shape to satisfy the zero moment and shear requirement. This change in shape requires a large deformation relative to that required by other types of load carrying members. The change in geometry of the cable allows it to carry the load. Such deflections constitute a second order effect. The system is nonlinear, and superposition of loads is not accurate as each load condition requires a unique geometry.

RESPONSE TO A POINT LOAD

A point load is perhaps the simplest load case for a cable.

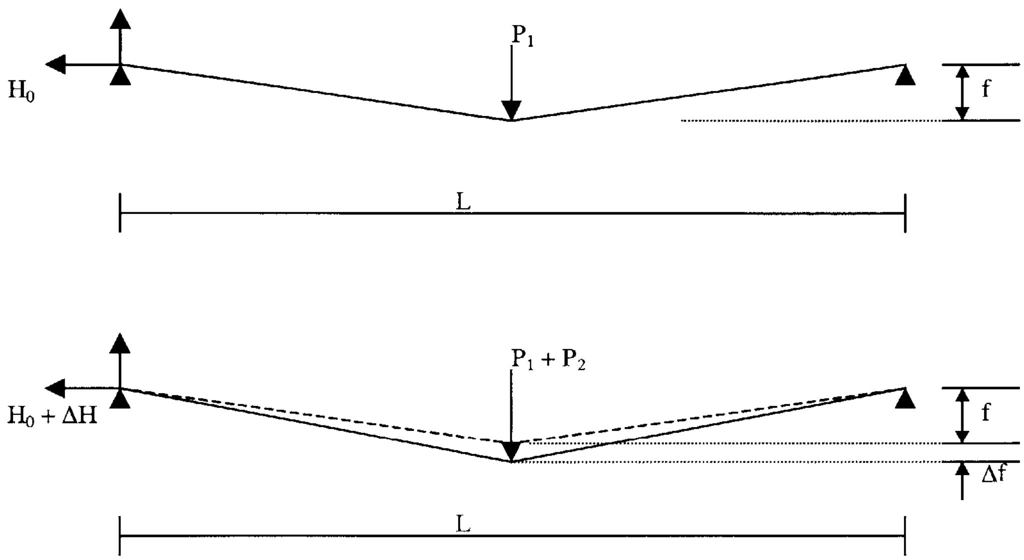


Figure 7: Deflection of a cable due to a point load.

Assuming cable under point load P_1 in the center of the span as shown in Figure 7, the horizontal force required is:

$$H_0 = \frac{P_1 L}{4f} \tag{1.}$$

With the addition of a second point load, P_2, the cable will elongate, resulting in a slightly different geometry and set of equations.

$$H_0 + \Delta H = \frac{(P_1 + P_2)L}{4(f + \Delta f)} \tag{2.}$$

or

$$(H_0 f + \Delta H \Delta f) + (H_0 f + \Delta H \Delta f) = \frac{(P_1 + P_2)L}{4} \tag{3.}$$

As the above equations show, the response to a second load involves two unknown quantities: the change in sag, and the change in horizontal force in the cable. These quantities are multiplied together in the solution, resulting in a highly non-linear solution.

Uniform Load

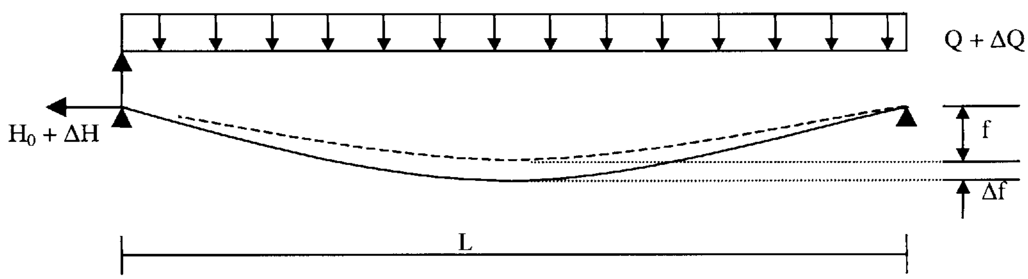


Figure 8: Deflection of a cable due to a uniform load.

Another simple loading paradigm is that of a uniform load applied to a cable. In this instance, the uniform load Q, is applied to the horizontal projection of the cable and not to the cable itself, as is the case for hung loads and live loads. The resultant horizontal force is:

$$H = \frac{QL^2}{8f} \tag{4.}$$

The addition of another load, ΔQ, results in:

$$H + \Delta H = \frac{(Q + \Delta Q)L^2}{8(f + \Delta f)} \tag{5.}$$

Horizontal Support Displacement

Support motion also causes deflection of the cable. Supports used in cable structures may be a compression ring, beams or tiebacks. No system is infinitely rigid, and therefore support motion must be examined. To examine the effect on a simple cable, a cable with a uniform load q is examined by moving one support a distance ΔL.

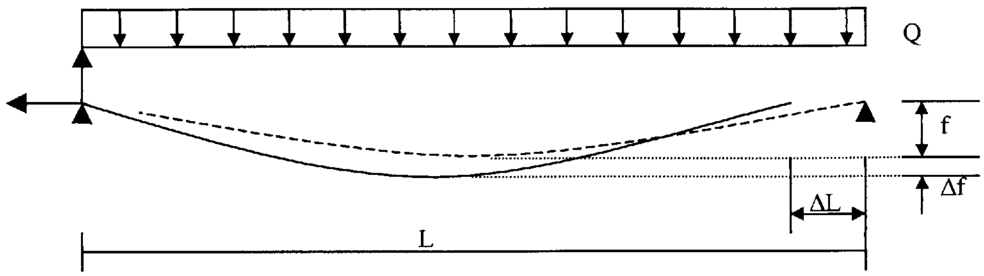


Figure 9: Displacement of a support of a simple cable.

The arc-length of a flat cable is approximately: (Szabo)

$$s = L(1+\frac{8}{3}\frac{f^2}{L^2}) \tag{6.}$$

The change in support position, ΔL, causes a corresponding vertical displacement Δf. Using an inextensible cable the arch length will remain constant:

$$\frac{\partial s}{\partial f}\Delta f + \frac{\partial s}{\partial L}\Delta L = 0 \tag{7.}$$

By substituting equation 7 into equation 8,

$$\frac{16}{3}\frac{f}{L}\Delta f + (1-\frac{8}{3}\frac{f^2}{L^2})\Delta L = 0 \tag{8.}$$

Assuming that $f^2/L^2 << 1$,

$$\Delta f \approx \frac{3}{16}\frac{L}{f}\Delta L \tag{9.}$$

Therefore, for cables with a low sag ratio (*f/L*), the increase in sag will be very large in proportion to the displacement of the support. If the sag ratio is around 0.094, the vertical displacement at the center of the span is approximately two times that of the horizontal displacement of the support.

3 CONSTRUCTION OF A CABLE NET

Construction of a cable net is inherently different from construction of most structural systems. A cable net is pretensioned and alters its shape significantly under loading. Furthermore, cables cannot be welded or bolted as other materials often are. These properties create two major obstacles, construction sequence and connection details, which are often unfamiliar to the standard-building designer.

3.1 CABLE PROPERTIES

Structural cables are manufactured in several forms including spiral strand and locked coil. Each form is a result of a different method of grouping wire strands which endows the final cable with different properties. Wire strands are made from very high strength steel with a Young's modulus of approximately 190 kN/mm^2 (Buchholdt). The alternative methods for wrapping wires allow for some modification of density, breaking strength, flexibility and resistance to deterioration.

For this project, spiral strand cables were assumed. These cables have a resulting modulus of approximately 170 kN/mm^2 for cables under 50 mm diameter and a tensile strength of 1.5 kN/mm^2. The values for breaking strength are not directly related to the cross sectional area of the cable by a fixed tensile strength as the strand arrangement varies slightly for every cable. For simplicity, the rough tensile strength will be used for design.

3.2 Connection Details

Cable nets are constructed with either a single or double cable system. In the former, the cables are connected by a U-bolt and a spacer, as shown in Figure 10. Alternatively, a double U-bolt system may be used as shown in Figure 11.

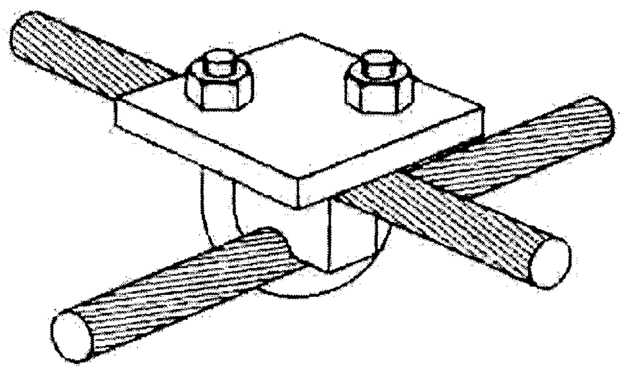


Figure 10: Single cable connection using U-bolt (Buchholdt pg. 271).

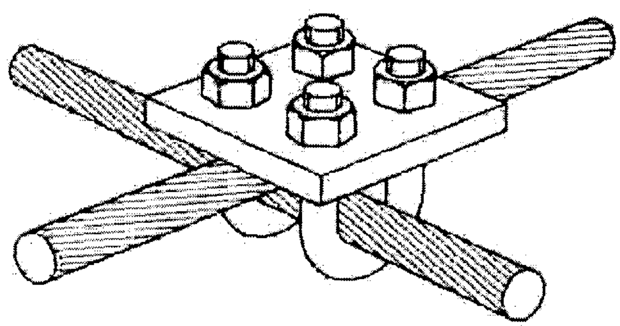


Figure 11: Single cable connection uing double U-bolt (Buchholdt pg. 271).

As cables become larger due to increased spans and loads, it may be desirable to use a double cable as opposed to a single cable. This results in the cable connections shown in Figure 12.

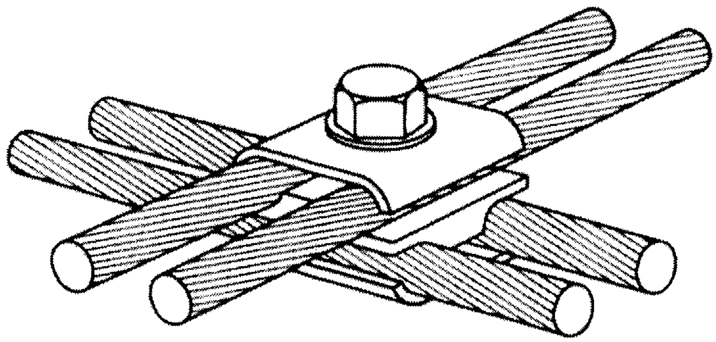


Figure 12: Connection for double cable configuration (Buchholdt pg. 271).

End sockets are usually designed such that they develop the full breaking load of the cable (Buchholdt). The end connections require the ability to be adjusted. This ability will be important primarily during the erection of the cables. Furthermore, during the life of the structure, the cables will have the tendency to creep and may require readjustment. A popular end detail is the bridge socket pictured in Figure 13. This detail is adjustable and allows for end rotation to avoid localized bending and fatigue effects.

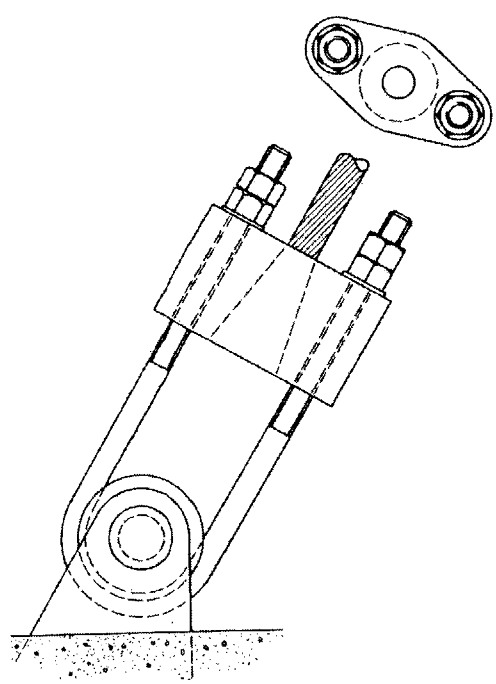


Figure 13: End connection detail (Buchholdt pg. 157).

Details similar to those described above were used on the Hong Kong Aviary. This aviary is constructed as an arch supported cable net. The net uses a dual-strand system to support a wire mesh cage roofing material.

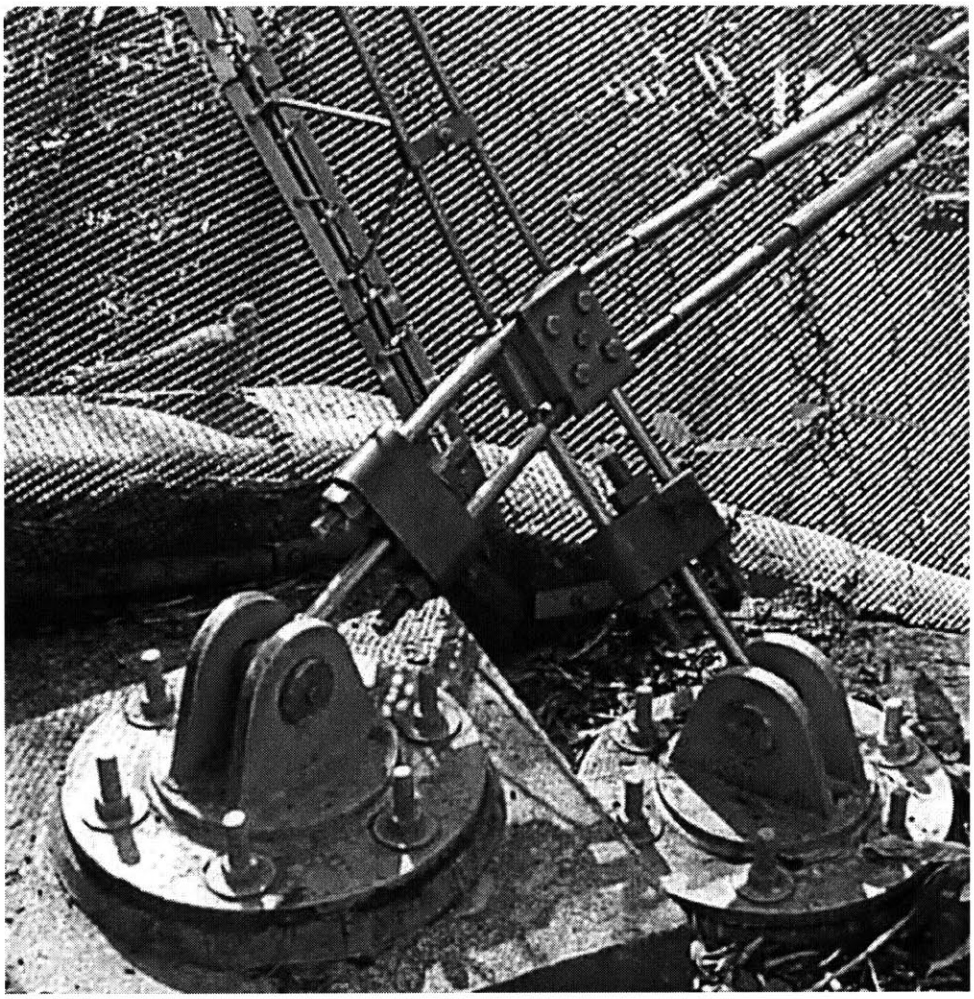


Figure 14: Connections at the Hong Kong Aviary.

3.3 Erection Sequence

The cables will elongate under any load, including the pretension load. Therefore it is important to carefully plan an erection sequence for a cable net. There are two basic methods for assembling a cable net. The net may be assembled on the ground and then lifted into place, or may be assembled in the air. For either method, a great deal of planning is required to insure that the process will proceed smoothly (Buchholdt).

Prior to erection, the cables must be linearized. Structural cables exhibit a non-linearity in their response to axial loading. Initially, the cables deflect significantly and do not return to the initial state upon unloading. This response is primarily due to the compacting of the strands within the cables as the strands shift. The resulting elongation is between 0.25 and 1% of the cable length and cannot be theoretically determined (Buchholdt). Therefore, it is necessary to prestress the cables prior to erection.

In the factory, the cables are subjected to the required load, which elongates them. The cables are then marked for length and shipped to the site. As the cables are placed and prestressed, they elongate back to the prestressed length measured at the factory. By this means, the cables are accurately pretensioned simply by fitting them into place.

4 Approximate Calculation of a Cable Net

The following describes a set of simplified calculations for a cable net roof. For this section, a procedure established by Szabo and Kollar was used. The calculations are approximate, and require numerous assumptions. The calculations by Szabo and Kollar were difficult to follow and contained a number of errors. Relevant formulas have been derived and several errors corrected for this chapter. A numerical example of this calculation is found in Appendix F.

4.1 General

The Szabo and Kollar equations are intended to be approximate calculations for a cable net on a compression ring. The compression ring is assumed to be elliptical. The compression ring is not a flat ring, but curves vertically as well to conform to the hyperbolic paraboloid shape of the cable net structure (Figure 5c).

The plan view of the system is shown in Figure 15. The important dimensions are the length and sag in the x and y directions. The cables which run in the y direction are set as the suspension cables and the x direction are the pretension cables.

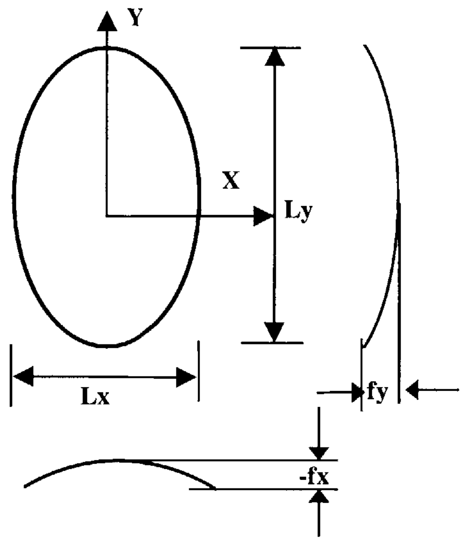


Figure 15: Plan view of structure to be calculated.

4.2 ASSUMPTIONS

To approximate the cable net, several assumptions were required:

- Only the central band of cables was examined in each direction. It is assumed that all cables parallel to each cable band exhibit similar forces and displacements.

- The net is considered to be flat. This assumption ensures that deflections in the x and y directions are zero. Furthermore, it simplifies calculations since $z'^2 \approx 0$.

- Loads are considered to be uniform and vertical only.

- Response of the cable net is considered to be linear. The principle of superposition will therefore be used. This assumption has been found to be approximately true for uniform loading in other literature (Buchholdt 1999).

4.3 Derivation of Approximate Calculation

The first step in this derivation is to separate the cable forces. The load is separated to establish the proportion of the vertical load to be carried by the pretension or suspension cable given each cable's stiffness characteristics.

$$qx + qy = Qtotal \tag{10.}$$

Where qx and qy represent portions of the load in the pretension and suspension cables. From equation 5:

$$H_0 f + \Delta H f + H_0 \Delta f + \Delta H \Delta f = \frac{(q_0 + \Delta q)L^2}{8} \tag{11.}$$

A change in tensile force on the cable:

$$\Delta s = \Delta H_x \frac{\partial s_x}{\partial x} \tag{12.}$$

Results in an elongation of the cable:

$$\Delta \partial s_x = \frac{\Delta H}{EA} \frac{\partial s_x}{\partial x} \partial s_x \tag{13.}$$

With:

$$(\partial s_x)^2 = (\partial x)^2 + (\frac{\partial z}{\partial x})^2 (\partial x)^2 \tag{14.}$$

And therefore, combining equations 13 and 14:

$$\Delta \partial s_x = \frac{\Delta H}{EA} \frac{(\partial x^2 + z'^2 \partial x^2)}{\partial x} = \frac{\Delta H}{EA} \frac{(\partial x \ + z'^2 \partial x \ )}{} \tag{15.}$$

equation 14, elongated by deflection *w*:

$$(\partial s_x + \Delta \partial s_x)^2 = (\partial x)^2 + (z' + w')^2 (\partial x)^2 \tag{16.}$$

$$(\partial s_x)^2 + \partial s_x \Delta\partial s_x + (\Delta\partial s_x)^2 = (z'\partial x)^2 + 2z'w'(\partial x)^2 + (w'\partial x)^2 + (\partial x)^2 \qquad (17.)$$

Subtracting equation 14 from this result:

$$2\partial s_x \Delta\partial s_x + (\Delta\partial s_x)^2 = 2z'w'(\partial x)^2 + (w'\partial x)^2 \qquad (18.)$$

Since, $\Delta\partial s_x << \partial s_x$:

$$2\partial s_x \Delta\partial s_x = 2z'w'(\partial x)^2 + (w'\partial x)^2 \qquad (19.)$$

$$\Delta\partial s_x = z'w'(\frac{\partial x}{\partial s_x})\partial x + \frac{w'^2}{2}\frac{\partial x}{\partial s_x}\partial x \qquad (20.)$$

Therefore, integrating and combining equations 15 and 20:

$$\int \Delta\partial s_x = \frac{\Delta H}{EA}\int_0^{Lx}(1+z'^2)\partial x = \int_0^{Lx}\frac{z'w'}{\sqrt{1+z'^2}}\partial x + \frac{1}{2}\int_0^{Lx}\frac{w'^2}{\sqrt{1+z'^2}}\partial x \qquad (21.)$$

And using the standard parabolic shape of a cable:

$$z = \frac{4f}{l^2}(lx - x^2) \qquad (22.)$$

And deflection of this cable:

$$w = \frac{4w_k}{l^2}(lx - x^2) \qquad (23.)$$

Where w_k is the increase in the cable sag. The flat cable approximation leads to:

$$1 + z^2 \approx 1 \qquad (24.)$$

The substitution of equations 22, 23 and 24 in equation 21 results in:

$$\frac{\Delta Hl}{EA} = \frac{16}{3}\frac{fw_k}{l} + \frac{8w_k^2}{3l} \qquad (25.)$$

Since:

$$\Delta H = \frac{\Delta q l^2}{8f}, \tag{26.}$$

The change in sag of the cable is:

$$w_k = \Delta H 3 \frac{l^2}{fEA} = \frac{\Delta q l^2}{8f} \tag{27.}$$

$$w_k = \Delta q \frac{3}{128} \frac{l^4}{f^2 EA} \tag{28.}$$

Setting the deflections of cables in x and y directions equal to each other and substituting a cable length for a distance between the supports by using the flat cable assumption:

$$q_x \frac{3}{128} \frac{l_x^3 s_x}{f^2 E_c A_x} = q_y \frac{3}{128} \frac{l_y^3 s_y}{f^2 E_c A_y} \tag{29.}$$

therefore:

$$Qy := \frac{Qd \cdot \left[\dfrac{Lx^3 \cdot sx}{fx^2 \cdot (Ec \cdot Ax)} \right]}{\left[\dfrac{Lx^3 \cdot sx}{fx^2 \cdot (Ec \cdot Ax)} + \dfrac{Ly^3 \cdot sy}{fy^2 \cdot (Ec \cdot Ay)} \right]} \tag{30.}$$

$$Q = Qx + Qy \tag{31.}$$

where Qd is the dead load, although any load may be used in this formula.

Since

$$\Delta H = \frac{\Delta q l^2}{8f}, \tag{32.}$$

The horizontal cable forces, n_x and n_y, are:

$$nxq := \frac{Qx \cdot Lx^2}{8 \cdot fx}, \quad nyq := \frac{Qy \cdot Ly^2}{8 \cdot fy} \tag{33.}$$

These results give the cable force in each direction due to the dead load *Q*. To obtain the required loading one can repeat the analysis using the snow, wind, and live loads, or simply scale the results to fit the proper loading. This is possible in the approximate calculation because of the assumptions of linear behavior and superposition. The results of this calculation will show that the suspension cables are in tension and the pretension cables are in compression. To overcome this inaccuracy, a pretensioning force equal or greater than the maximum compression force must be applied. The pretension forces in the x direction cables will result in a pretension force in the *y* direction cables calculated by:

$$nyp = nxp \frac{f_x}{f_y} \frac{l_y^2}{l_x^2} \tag{34.}$$

and the vertical deflection at the center point is calculated by:

$$w_p = \mathrm{nxp} \frac{3}{16} \frac{\mathrm{l}_x^2}{\mathrm{f}_x \, EA_x} \tag{35.}$$

The end deflection is therefore the sum of the deflection due to the maximum load and the deflection due to the pretension load. Since the system is assumed to be linear, loads and deflections may be scaled. The design example calculations are shown in Appendix F.

4.4 Results From Infinitely Rigid Compression Ring Calculations

The analysis using an infinitely rigid compression ring produces a first estimate of the size of cables and expected deflections. In this analysis, the variables which may affect the stiffness are limited to the cable properties and the cable sag.

The analysis uses a system of variables which does not easily lend itself to optimization. However, an increase in sag of the cables results in a greater length, but a decreased force in each cable. Therefore, this property may be optimized to reduce the total material in the cables. There are other parameters which may affect the design such as limiting the deflection of the structure under loading. The structure could be limited in a standard fashion, to an L/240 requirement, increasing the complexity of the problem.

4.5 Application to the Rockwell Cage

Design loads are detailed in Appendix B and include dead load (*Qd*) snow load (*Qs*) and wind load (*Qw*). For the Rockwell Cage, the results show that for the given load cases, an incredibly small amount of steel is required. The resulting cables are 15.6 mm^2 and 217 mm^2, with a total deflection, due to dead, snow and pretensioning loads, of 0.562 meters upwards. Deflection due to the live load only (snow) is 0.343 meters downwards or L/140 on the long span. The design value of L/240 results in a 0.197 meter allowable deflection under snow load which is equivalent to a 0.097 meter allowable deflection under dead load (*wx*). The stiffness of this structure must be increased to meet the desired values, or the design requirement must be changed. The methods of achieving an increase in stiffness include adjusting the cable areas and cable sag values.

The following graphs show the effect of changing various parameters on the deflection, and thus stiffness, of the system. The charts were created using the approximate calculation described above and the following values:

Length of Structure	Ly	60 Meter
Width of Structure	Lx	48 Meter
Area of Suspension Cables	Ay	2.604 e^{-3} $Meter^2$
Area of Pretension Cable	Ax	2.34 e^{-4} $Meter^2$
Sag in Suspension Cable	fy	6.026 Meter
Sag in Pretension Cable	fx	-2.4384 Meter
Dead Load	Qd	704 Pascal
Snow Load	Qs	1436 Pascal
Wind Load	Qw	-603 Pascal

The response of the cable net to an increasing load is linear according to these equations (Figure 16). This is the result of the initial assumptions and is therefore expected.

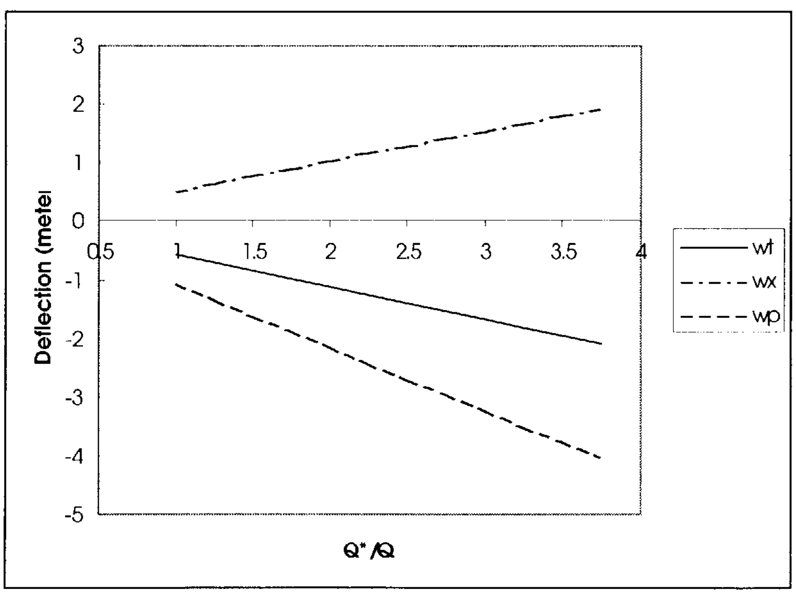


Figure 16: Results from approximate calculations of cable net on rigid ring with varying load.

The graphs below show that there is an ability to alter the stiffness by various means. One solution which achieves the desired stiffness characteristics is simply to increase the suspension cable area. This solution requires no change in the other variables, and results in the desired performance characteristics.

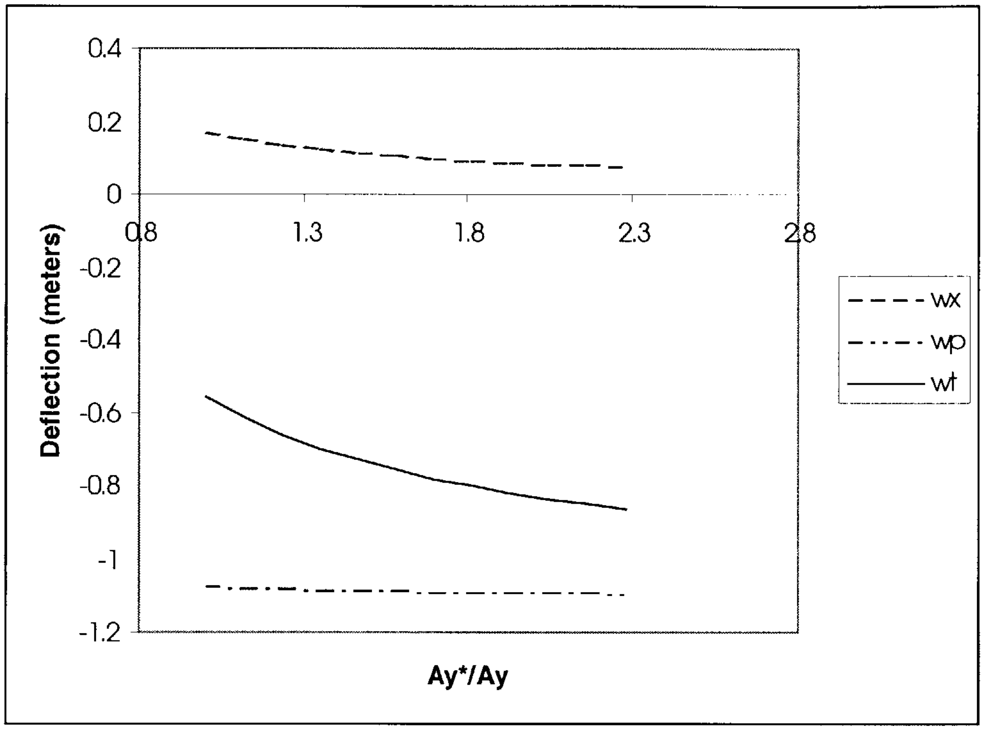


Figure 17: Effect of Ay on deflection for the approximate calculation of the cable net with a rigid edge.

When the area of the pretension cable is increased as some load, the pretension force is determined by the unloading of the suspension cable instead of the unloading of the pretension cable.[1] This accounts for the discontinuity of the curve. In general, the increasing of the pretension cable area also decreases deflections.

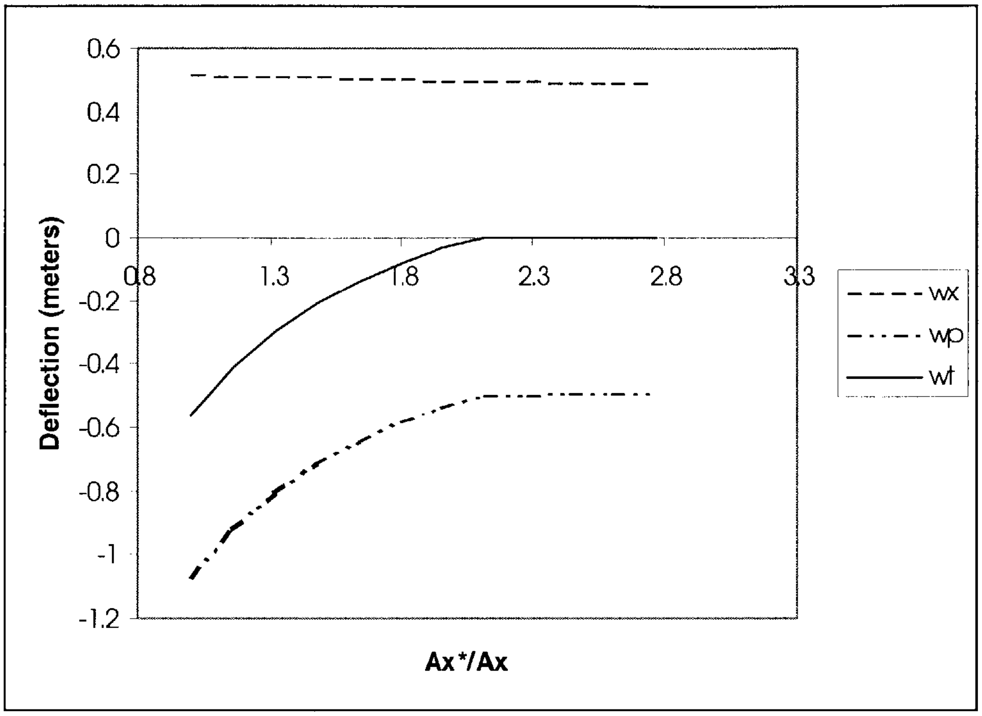


Figure 18: Effect of Ax on deflection for the approximate calculation of the cable net with a regid edge.

---

[1] This is also true as the area of the suspension cable is decreased, however since this would exceed design limits, it was not shown.

A similar result is found with the increase in pretension cable sag, fx (Figure 19). The structure increases in stiffness with the increase in cable sag. The relationship is non-linear.

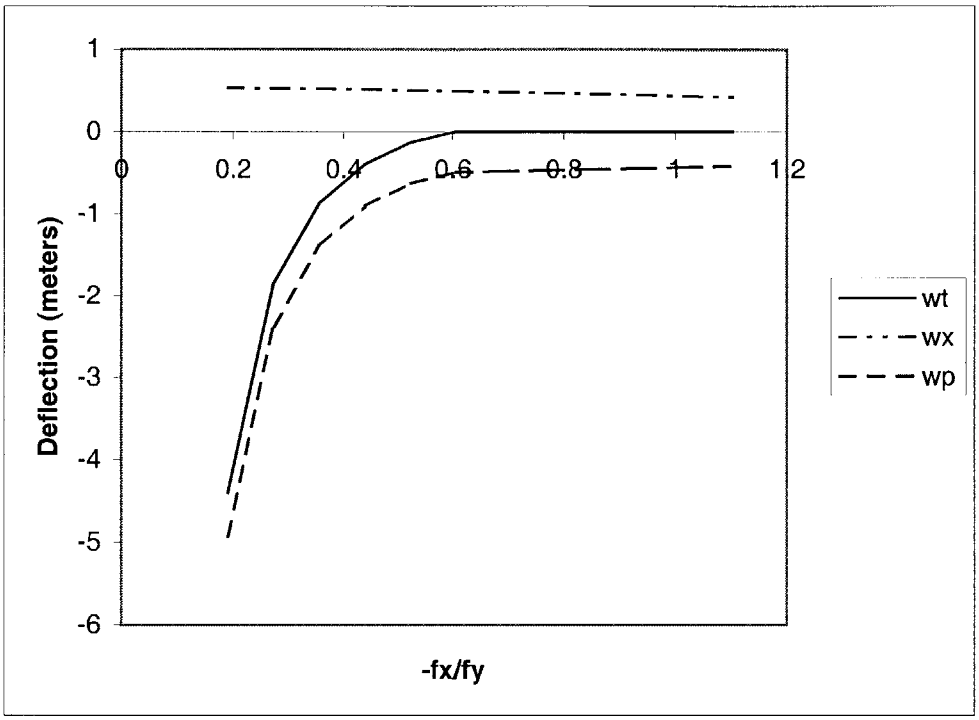


Figure 19: Effect of fx on the approximate calculation of a cable net with a rigid edge.

4.6 INTRODUCTION OF THE FLEXIBLE COMPRESSION RING

The previous analysis assumed that the edge ring is infinitely rigid, which will never be the case. Since small support movements can result in large mid-span deflections, the flexibility of the edge ring should be considered. For this analysis, it is assumed that the compression ring deforms in bending only and that the deflection due to the compression forces are negligible. Furthermore, the compression ring is assumed, for the sake of analysis, to exist in a flat plane. This is acceptable since the ring is fully supported in the direction perpendicular to this plane.

The primary technique used to analyze this problem is to separate forces which cause pure bending from those which produce pure compression. The forces which produce pure compression are those which act inwardly from all sides. The forces which act inward on one set of cables, and outward on the other are those which cause pure bending (Figure 20). In the calculations to follow, the forces which cause pure bending will be labeled with a 2 and those causing compression labeled with a 1.

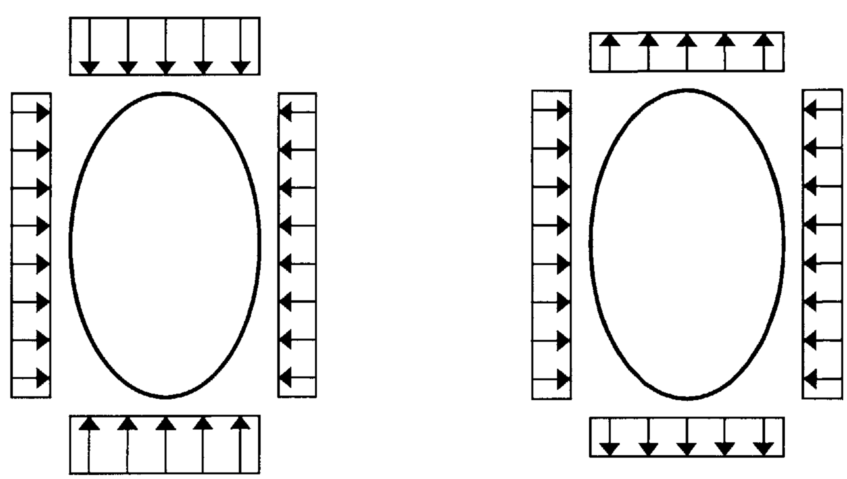


Figure 20: Forces causing pure compression (left) and pure bending (right) in a ring.

The equations used in these calculations are from Szabo, and may be seen in the example solution in Appendix G. These calculations are added for completeness and not derived in this paper.

As expected, the introduction of the compression ring alters the results of the calculation significantly. It was shown in section 2.1 that a deflection of a support substantially affected the deflection of a cable. As the structure is loaded, the suspension cables cause the compression ring to deflect, shortening the distance between ends of the suspension cables and outwardly deflecting the ends of the pretension cables. Therefore, as the ring deflects, a greater force in the pretension cables is induced.

Another aspect of these calculations becomes interesting following the introduction of the compression ring. The initial sag in the cables greatly affects the bending moment in the compression ring. As is shown in the calculations, the pretension force may be calculated such that it minimizes the total design moment for the compression ring. As long as this pretension force is also greater than the minimum force required to keep all of the cables in tension under all design loads, it is an acceptable solution. Szabo and Kollar's calculations show that the two values match at approximately $fx/fy = 0.53$. Therefore, at this point the structure would be optimized for bending stresses in the compression ring. Following their calculations exactly, it is found that the 0.53 ratio for fx/fy holds true for the Rockwell Cage structure as expected (Figure 21).

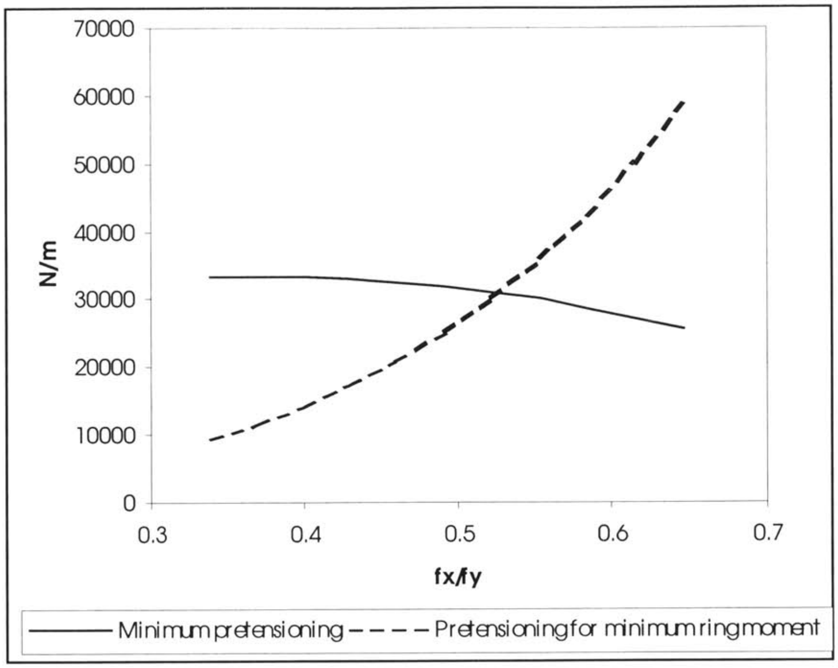


Figure 21: Results for Szabo's caclulation on Rockwell Cage

This ratio does not hold true when the calculations are applied to the Massachusetts State Building Code load factors. The Szabo and Kollar calculations use the maximum uplift condition of wind in the absence of dead load while the local code uses a portion of the dead load in conjunction with the wind load. This reduces the maximum uplift condition significantly. Therefore, the required curvature on the pretension cables is not as great in the optimized condition. The optimum value for fx/fy, to reduce the bending moment of the compression ring, occurs at approximately 0.14 (Figure 22). If the suspension cable is set at a sag-span ratio of 0.1, as has been done for these calculations, the resulting sag ratio in the pretension cable is therefore only 0.02 for the Rockwell Cage. This value is extremely small and raises concerns of flutter and other local instabilities. Buchholdt, for example, recommends a minimum cable sag/span ratio of 0.04 to avoid local effects (Buchholdt). Furthermore, a minimum curvature is required to prevent ponding and allow for sufficient drainage. This is perhaps the most significant reason for setting a minimum curvature of the pretension cable. Although these criteria need further research and analysis to be set correctly, a minimum of 5% sag in the tension cable will be used resulting in a *fx/fy* ratio of approximately 0.4.

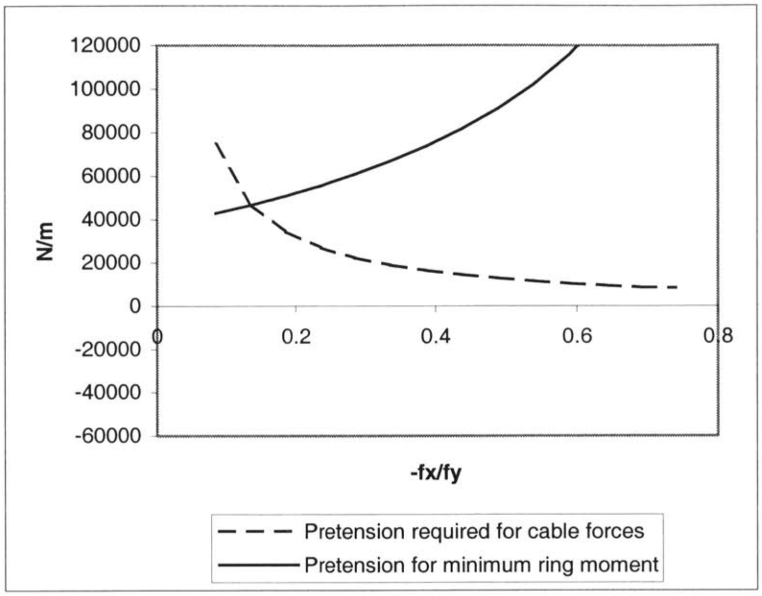


Figure 22 Results using modified analysis for Massachussets State Code

With the value of sag in the two cables set, the pretension required for the two optimal situations may be found in Figure 22. It is therefore clear that the pretension required to avoid a loss of tension in the cables is far less than that required to obtain the minimum design ring moment. Due to the large difference between the two values, it will be assumed that the compression ring will be designed to resist the required moment even though it is not the absolute minimum. This analysis is clearly simplified, and a cost analysis of this question would be beneficial to determine the validity of choosing between the values.

The parameters that affect the cable net with the ring are the same as those affecting the net without the ring, with the addition of the compression ring moment of inertia. The moment of inertia of the compression ring results in a significant decrease in stiffness which must therefore be compensated for in other aspects of the structure (Figure 23).

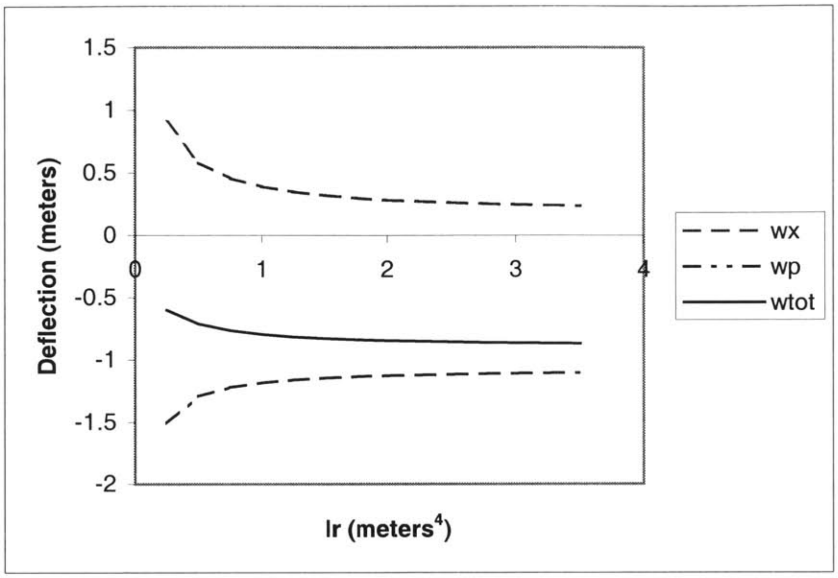


Figure 23: Change in deflection with respect to Ir

The equations presented by Szabo and Kollar represent a useful guide for a preliminary design of a cable net structure. In using these equations and the subsequent conclusions, however, it is important to note the effects of various parameters. The system of equations required to estimate the behavior of a cable net roof is extremely complex and indeterminate, resisting efforts to draw simplistic conclusions. In determining the guide ratio for minimum bending in the compression ring it is crucial to determine ring stiffness and load criteria.

The above calculations are able to offer some direction, but they do not fully explain the cable net system. Cable structures behave in a non-linear fashion. The above analysis is unable to calculate system response to non-uniform loading and is sure to have errors due to the linear approximation.

5 Computer Analysis

5.1 SAP 2000

SAP 2000 Non-Linear seemed to be the perfect tool to analyze a non-linear structure. The program is well suited for a p-delta analysis of a regular structure. However, the program is be unable to correctly analyze cable members. If the members are released at the ends, the structure quickly becomes unstable. The program also fails to analyze tension only members, thus allowing them to carry compression loads.

5.2 Analysis using Easy

SAP 2000 exhibited significant shortcomings in the analysis of cable net structures. To achieve a more accurate analysis, the program EASY, by Technet, was employed. Technet is a German company which specializes in lightweight structural design and consulting. Their program is self described as "a fully comprehensive suite of software modules for the complete design of geometrically non-linear lightweight structures (EASY 2000)."

The program is efficient at finding the appropriate form for a given set of constraints such that cables are in tension and forces are in equilibrium. This process of form-finding is otherwise difficult and has involved various techniques including soap bubbles and hanging sheets (Otto). The process is greatly facilitated by EASY.

Despite its name, the program was less than straightforward. The program is a set of individual sub-programs, each of which performs a specific function and therefore must be run sequentially to obtain meaningful results. It is not immediately apparent what each of the sub-programs does, and took much research of the help menus to determine which to run and in what order.

The primary function of the program may be its form-finding ability. The following example form was found in less than five minutes. This example shows a composition of a net hung from a series of random fixed beams. While this particular case is not applicable to the Rockwell Cage design, it displays the power of such a program in determining net shapes for almost any set of parameters.

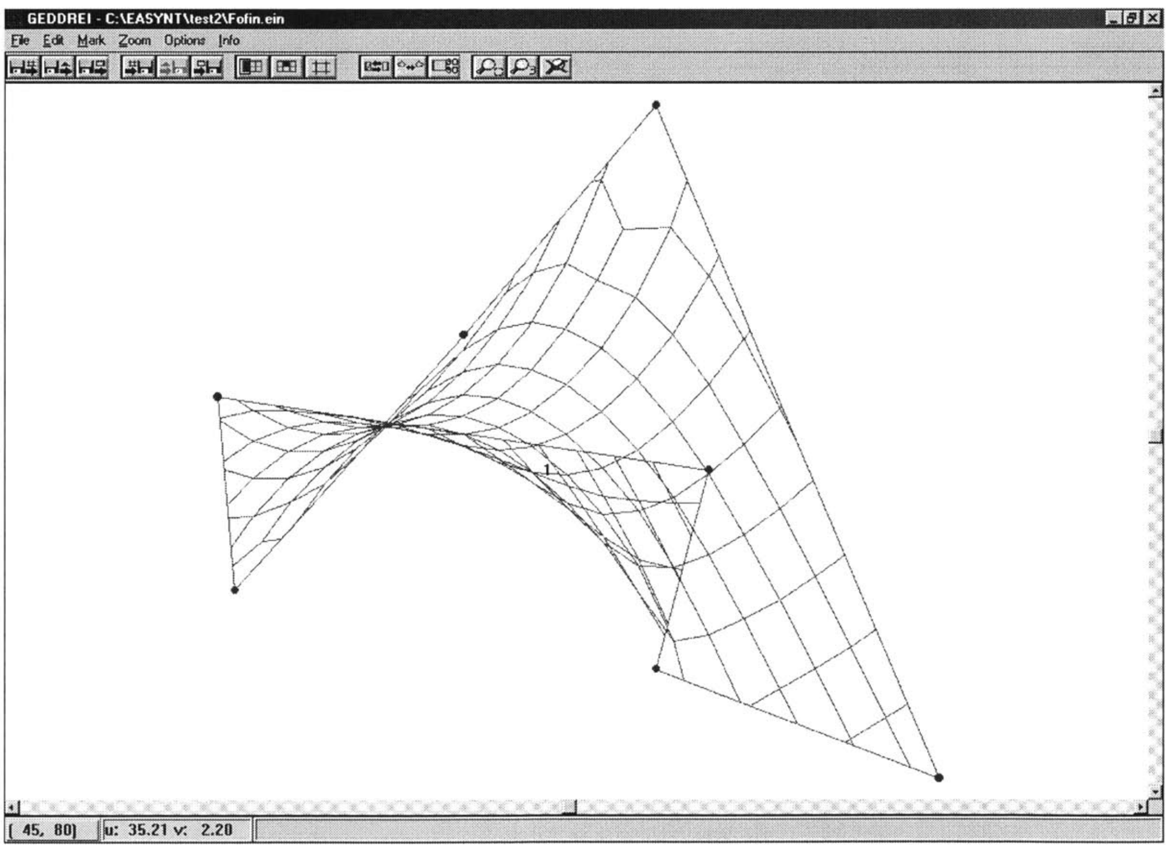


Figure 24: Random beam net formation using EASY.

Applying the program to a set shape and defined parameters is a more difficult matter. The program does not allow parameters such as cable areas and Young's modulus to be specified. The results could not be used since the program also appeared to be dimensionless, and therefore the default value was unknown. However, a simplified model for the Rockwell Cage was constructed as shown in Figure 25.

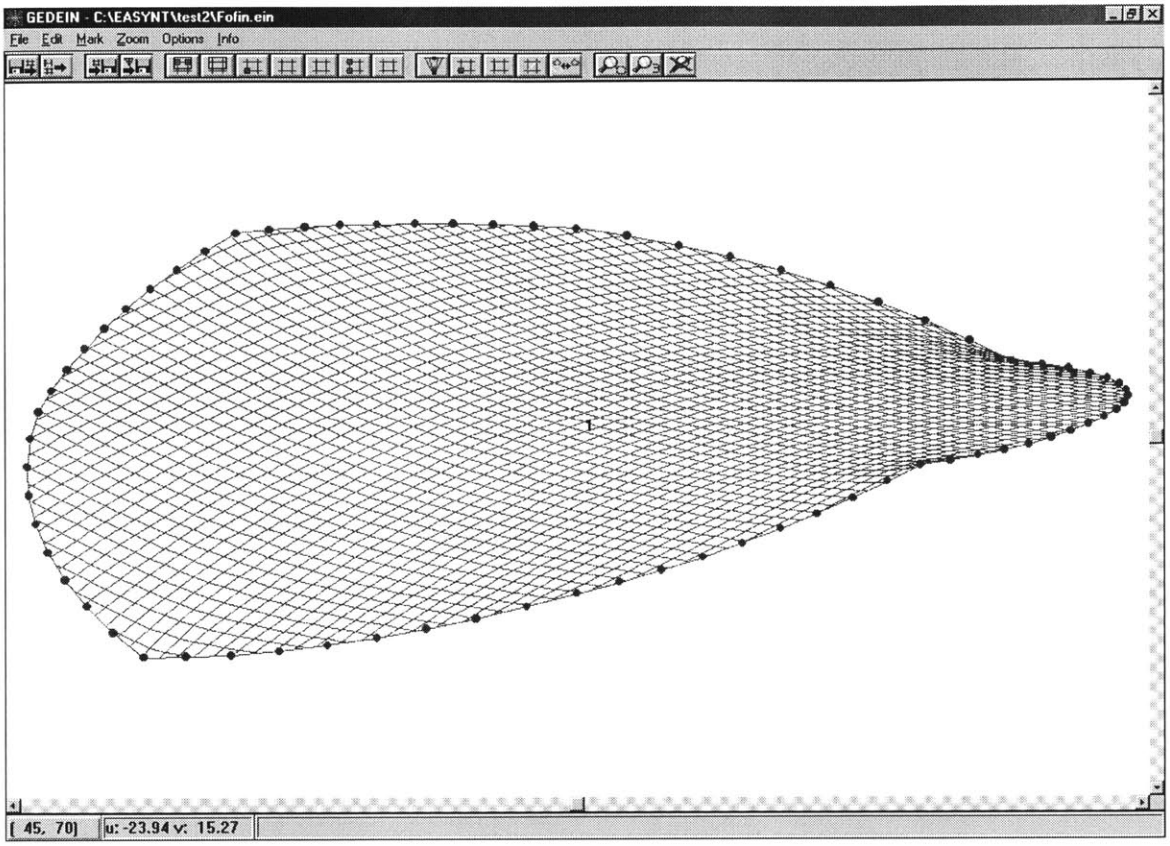


Figure 25: EASY mesh generaion for simple model of Rockwell Cage.

EASY was written with the intent of form-finding, calculating stresses and quickly determining a cut-out pattern to efficiently construct the membrane. Because it does not allow the input of Young's modulus or cable areas, EASY is not an effective tool for finding deflections and examining stiffness.

6 NON-LINEAR MATRIX ANALYSIS

The failure of alternative structural analysis programs to offer desired results led to the creation of a non-linear matrix solution. Such a solution may not be as complete, or user-friendly, but allows for a much better understanding of the parameters and problems involved in solving cable nets while offering and exact solution. The analysis of these structures is simplified by the nature of cables and the loading pattern. Since the loads are applied at the cable joints, and the cable only carry axial loads, a simple rod equation system may be used. The stiffness equation has two parts, a linear stiffness (K_l) and a geometrical non-linear stiffness (K_g). The terms from each are described below.

$$K_t = K_s + K_g$$
$$K_l = \frac{AE}{L}\alpha^t\alpha \qquad (36.)$$
$$K_g = \frac{F}{L}I$$

Where:

$$\alpha = (\frac{x_2 - x_1}{L_1}, \frac{y_2 - y_1}{L}, \frac{z_2 - z_1}{L}) \qquad (37.)$$

And finally:

$$U = K^{-1} * P_e \qquad (38.)$$

The solution of these equations involves an iterative method. First, the equations are solved as written above and deflections are found. Next the stiffness matrix is adjusted to account for the change in position of the members. This adjustment includes the change in force in the members due to their elongation and the change in alpha due to the deflection. An internal load vector is subsequently created to account for the forces in the members. This new vector is added to the external load vector and the equations are solved.

$$\begin{aligned}\Delta x &= (x_2 + u_2) - (x_1 + u_1) \\ \Delta y &= (y_2 + v_2) - (y_1 + v_1) \\ \Delta x &= (z_2 + w_2) - (z_1 + w_1)\end{aligned} \tag{39.}$$

$$L_2 = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \tag{40.}$$

$$\beta = (\frac{\Delta x^2}{L_2}, \frac{\Delta y^2}{L_2}, \frac{\Delta z^2}{L_2}) \tag{41.}$$

The new length can be used to determine the force in each member:

$$\begin{aligned}e &= L_2 - L_1 \\ F_2 &= \frac{EA}{L_1} e\end{aligned} \tag{42.}$$

Internal forces and the stiffness matrix can then be found:

$$P_i = F_2 \beta \tag{43.}$$

$$K_t = \frac{AE}{L_2} \beta^T \beta + \frac{F}{L} I \tag{44.}$$

And finally:

$$\Delta U = K^{-1} * (P_e + P_i) \tag{45.}$$

$$U_{next} = U + \Delta U \tag{46.}$$

The solution for the deflections can then be entered into equation 39 and the process repeated until a solution is found. A solution is found when external forces balance internal forces or the change in the displacements is zero.

6.1 CREATION OF THE COMPUTER PROGRAM

The program was written using Matlab. The general flow of the program can be seen in Figure 26. The program begins by reading the data. At this point, the two data files are read, supplying joint locations, boundary conditions, connectivity and cable areas. Then initial values are calculated including member lengths, alpha, and member pretension forces. The first incremental value for displacement is then calculated. Using this value the program enters the main loop. New lengths, forces and alpha are calculated which results in a new value for the displacements. The solution is then checked for convergence. If the solution has not converged, the program stays within the main loop until the solution converges. Once the program has converged, the initial values can be changed in the larger loop, allowing for examination of the main parameters. The code may be found in Appendix C.

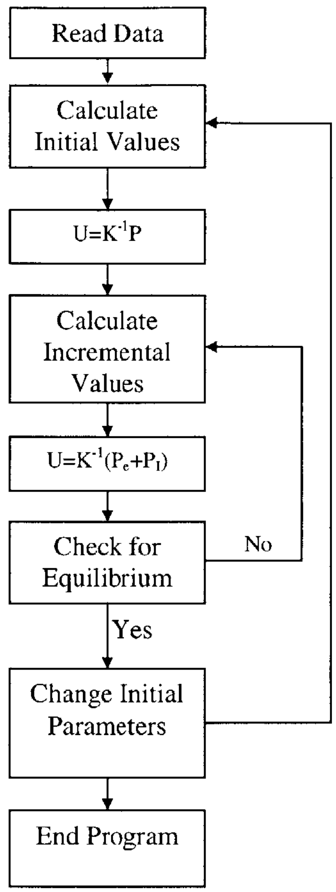


Figure 26: Flow of the computer program.

6.2 Solution of the Flat Cable

The flat cable is an interesting problem which cannot be solved by most simple structural design programs. However, the analysis described above effectively finds a solution. Using the computer program formulated for Matlab, the analysis is straightforward. The analysis quickly converges, requiring less than twelve iterations for most cases. The solution of the flat cable served as a check of the program as well as an interesting study. Given a particular solution, it is a simple process to check that equilibrium has been achieved.

The program was run on a flat cable with two rod elements each with a length of 30 m, an area of 1 m^2, and an initial pretension of 10 N. Young's modulus was set at 1.6 e ^{11} Pa. Under a point load of 1 Newton, the solution found was that the center point moved only in the vertical direction of the amount –5.519 e $^{-3}$ m. Therefore, the elongation of the cables is $e = \sqrt{30^2 + (-5.519e^{-3})^2} = 5.0766e^{-7}$ m, and the force in the cable is $F = \frac{AE}{L}e = 2717.5$ N. The vertical component of this force is 0.5 N. Since there are two cables, the total vertical force is therefore 1.0 N, and the system is in equilibrium. A similar test was run using four cables.

To understand the parameters that affect the response of a cable, each was altered and the response was observed. The Matlab solution was ideal for this study as a simple loop could be inserted to change parameters and record the response. Running such a study on most commercially available software would have been far more tedious.

The first analysis examined the response of the cable to a changing point load. The load was increased from -1 to 8.8 N. The response shown in Figure 27 demonstrates the non-linear response of the cable. As load is increased, the incremental deflection decreases. Therefore, the cable increases in stiffness under increasing load. At low loading the stiffness of the cable is much less, resulting in the significant deflections near zero load.

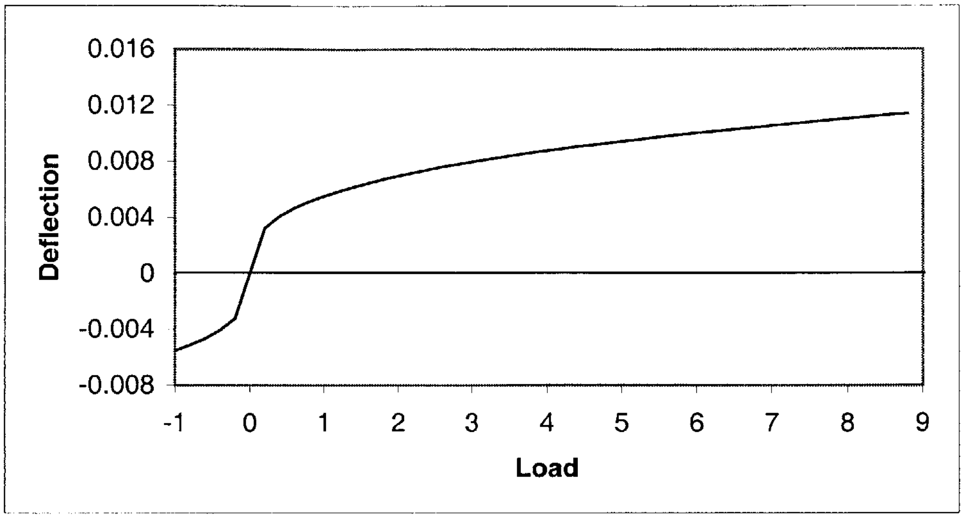


Figure 27: Flat cable subjected to a varying point load.

The final geometry of the flat cable is dependent upon the pretension force. This can be seen from the equations of the cable, as the initial linear stiffness matrix is zero with respect to vertical deflections. Deflections vary linearly with the pretension force as can be seen in Figure 28.

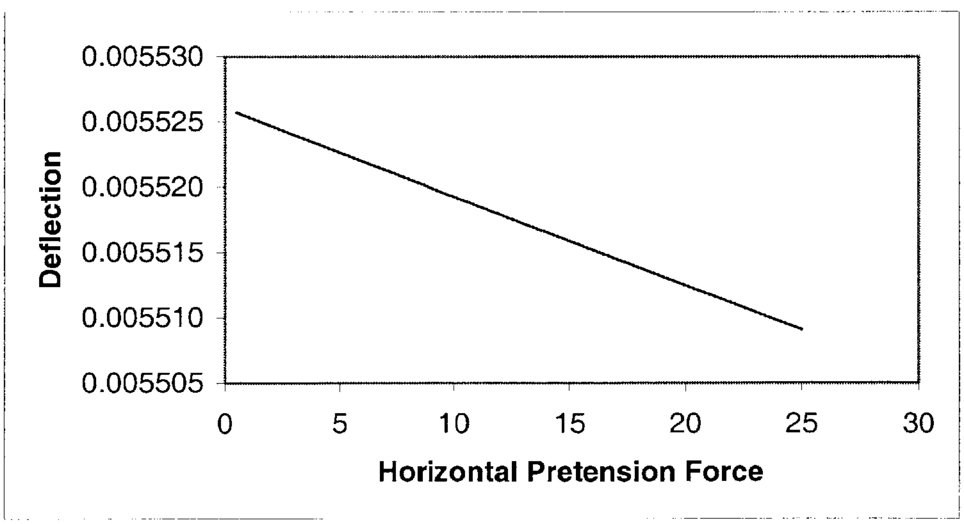


Figure 28: Cable subjected to a point load with varying pretension force.

The cable exhibits a non-linear response to a change in cable area. This non-linear response is due to the increase in restoring forces as the area increases. The response of various size cables to a point load is shown in Figure 29.

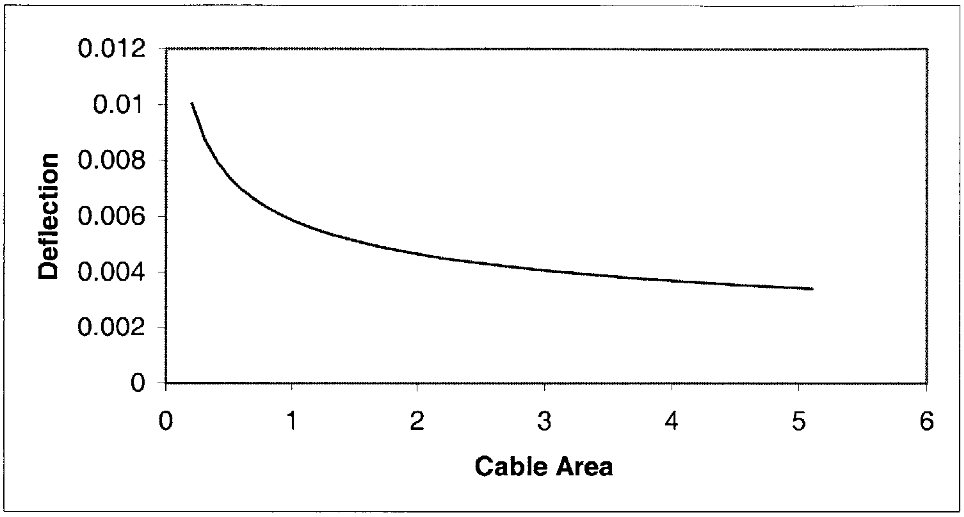


Figure 29: Deflection of a cable subject to point load with varying cable area.

The final study performed on the flat cable was an increase in initial sag. The effect of the initial sag is an increase in both the linear and non-linear stiffness terms. Furthermore, the non-linear term has a vertical internal force as a result of the geometry and the pretension force.

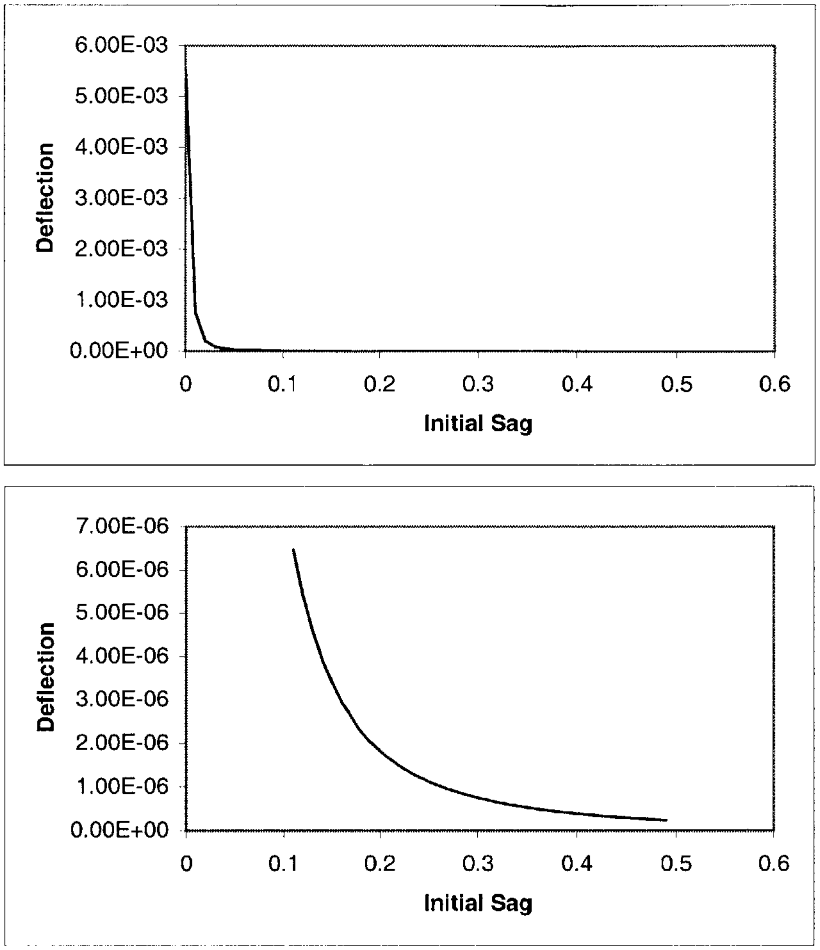


Figure 30: Cable subjected to point load with a varying initial sag.

6.3 Two Opposing Curvature Cables

This analysis can be expanded to include two cables with opposite curvatures. In this case, the structure is made up of four rods to create an X formation. The top two cables are the suspension cables as they are correctly positioned to carry the load. The bottom two cables are the pretension cables. They allow the structure to be pretensioned while retaining its shape. The structure is more rigid in both directions than the plain cable as the system has the added rigidity supplied by the pretension force.

The following system was tested using the Matlab program. The cables have areas of 1 m^2, horizontal lengths of 10 m and sags of 1 m. A Young's modulus of $2{*}e^{11}$

Pa was used, and the load varied between 10 and 100 N. The results are shown in Figure 31.

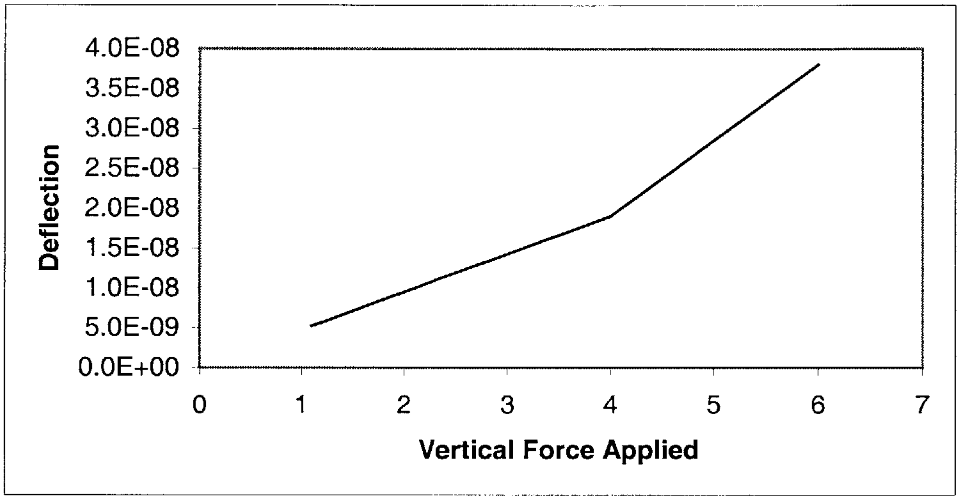


Figure 31: Displacement of a two cable system.

The results for this system demonstrate the system's initial rigidity. Deflections on the system are relatively small compared to a comparable cable. Of primary interest is that as the load is increased, it becomes sufficient to remove tension in the pretension cables. Under this load, the system stiffness decreases. More importantly, if the system were loaded to such a level, the decreased stiffness, particularly in the opposite direction, may allow for instability in regards to wind loads which may cause flutter.

6.4 Cable Net

Having proven the efficacy of the program, and examining parameters of simple systems, a cable net with proportions roughly that of the Rockwell Cage was examined. The model used was a rectangular grid with a total area of 48 meters by 60 meters. Three cables span in each direction, following the geometry of a hyperbolic paraboloid of

$z = \frac{4f_x}{L_x^2} x^2 + \frac{4f_y}{L_y^2} y^2$. Initial values were obtained from the equations from Szabo and Kollar described previously. Pretension forces, cable areas and loads were scaled to reflect the much greater cable spacing for this model.

Spacing of Suspension Cables	Dy	12 Meter
Spacing of Pretension Cables	Dx	15 Meter
Area of Suspension Cables	Ay	2.604 e^{-3} Meter^2
Area of Pretension Cable	Ax	2.34 e^{-4} Meter^2
Horizontal Pretension Applied to Suspension Cable	Hy	214,672 Newton
Horizontal Pretension Applied to Pretension Cable	Hx	325,165 Newton
Sag in Suspension Cable	fy	6.026 Meter
Sag in Pretension Cable	fx	-2.4384 Meter
Uniformly Applied Point Load	P	385,267 Newton

The results from this example were surprisingly different from the results predicted by Szabo and Kollar. The deflection of the center point of the net was found by matrix analysis to be 3.2 meters. The expected deflection by approximate methods was 1.507 meters upwards due to the pretension force and 0.504 meters downward due to the combined loading, resulting in a 0.569 meter deflection upwards. The Matlab model had a negligible deflection as a result of the pretension forces alone.

The difference in pretension deflection is a result of the method of solution. Szabo and Kollar treat the pretension force in the same manner as a gravity loading. The force is converted into an equivalent static load, which counteracts the applied load. Therefore, a significant deflection is expected. The Matlab solution assumes that the cables are elongated but under stress. If the initial geometry is exact for the given load, the initial deflection should be zero. For this case, the pretension force caused an insignificant deflection, but it did cause some deflection. Increasing pretension forces increased the deflection, but the total pretension deflection remained negligible with respect to the total deflection, demonstrating that the form was correct.

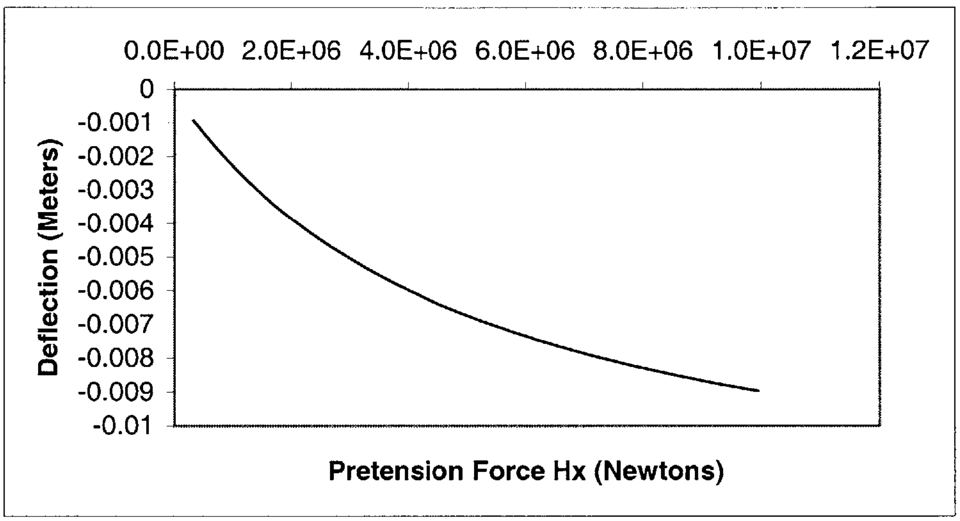


Figure 32: Deflection due to pretension force alone.

(Note negative deflection implies upward motion.)

Another difference between these two calculations is that the Matlab solution shows an unloading of the side pretension cables under load. This may be due to the rectangular geometry as opposed to the elliptical edge ring. Such a ring would add stiffness to the cables that are off center, as they become shorter. Although the center pretension cable is still active, the loss of tension in the side cables significantly affects deflection. In order to examine this, the horizontal pretension was varied to observe the effect of the unloading of these cables (Figure 33). The stiffness is nearly linear to the point where all cables remain in tension. At this point, the structure increases in stiffness dramatically, remaining nearly linear.

Szabo and Kollar calculated deflections and forces for the central cables only. Using the Matlab program, this approach is found to be invalid for a rectangular plan. While the central cables exhibited roughly the same forces as calculated approximately, the unloading of the side cables significantly impacted the response of the structure. Had the cables remained in tension for the matrix analysis, it is assumed that the model would have responded with a deflection of approximately 1.8 meters (Figure 33). Such an approximation is based on a tangent stiffness value, which assumes a linear response. This assumption of linearity appears to be nearly correct. However taking a different tangent value, it can be seen how a deflection value of 0.5 m can be found. Szabo and Kollar assume that the stiffness is linear for their calculations, which results in significant inaccuracies for this particular case.

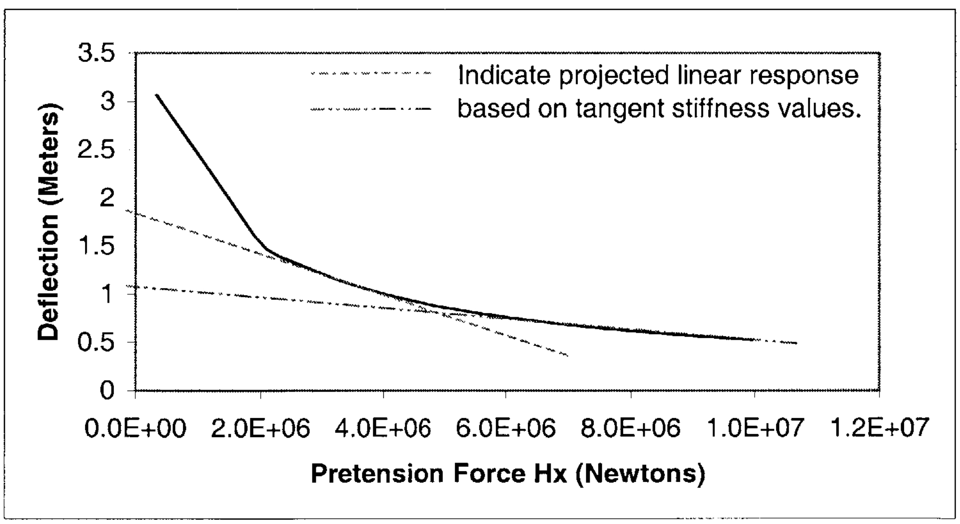


Figure 33: Deflection of cable net with varying horizontal pretension.

(Hy = Hx/1.8636)

The final parameter examined was the area of the cables. With increasing area of cables, a decrease in deflection is observed as expected (Figure 34). It is clear that the system can be solved, and the parameters behave correctly, but that the system is highly complex and non-linear. An appropriate solution would require optimization of the various parameters, which is beyond the scope of this study.

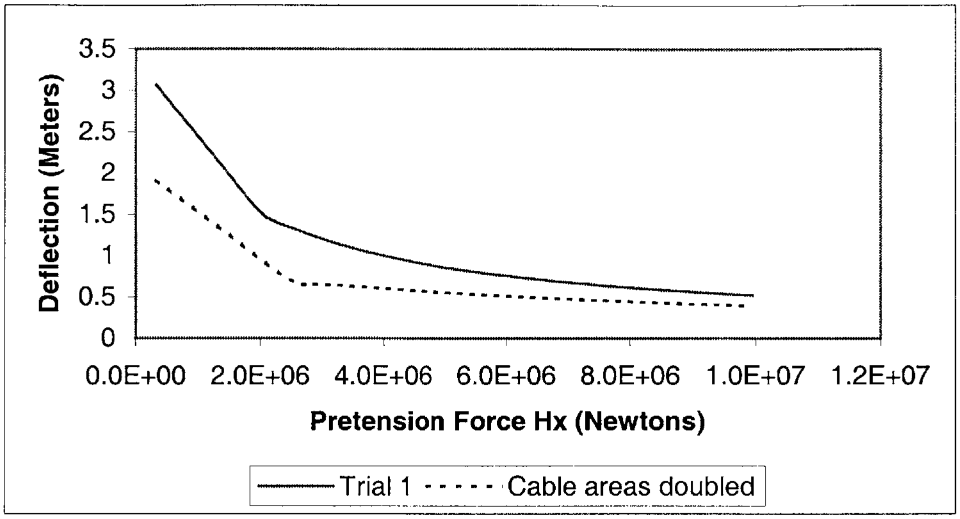


Figure 34: Deflection of cable net for two trials.

7 Conclusion

Cable net structures offer a pleasing form with unique structural properties. However, many designers and most standard design tools are not equipped to design these structures. Approximate solutions, are inadequate for examining the behavior of cable structures.

A non-linear matrix analysis is a proper tool for analyzing a cable structure. It provides the ability to analyze any form, under any loading. It correctly handles tension only members, and can be modified to include compression and bending members.

It is apparent that even with such a tool, designing cable net structures remains a difficult task. The behavior of a cable net depends on a large number of parameters and behaves non-linearly. A proper engineering solution to such a problem would involve optimization of all of these factors.

Bibliography

Architectural Fabric Structures: The Use of Tensioned Fabric Structures by Federal Agencies. National Academy Press, Washington, *D.C. 1985.*

Blau, Stacey E. "MIT Maps-out Plan for Athletic Center," *The Tech.* Volume 116, Number 63, December 3, 1996.

Bucchholdt, *H. A. An Introduction to Cable Roof Structures.* Thomas Telford Ltd. London. 1999.

"EASY: A System for Lightweight Structure Design from Technet GmbH." from Technet GmbH Homepage http://www.technet-gmbh.com/english_old/easy_e.html accessed 3/29/2000.

Irvine, H. Max. *Cable Structures.* The MIT Press. 1981.

Long Span Roof Structures: Proceedings of a Symposium Held at the 1981 Annual Convention and Exhibit St. Lous, Missouri, October 26-30, 1981. American Society of Civil Engineers, New York, 1981.

The Massachusetts State Building Code: 780 CMR Sixth Edition. State Board of Building Regulations and Standards, 1997.

Minimum Design Loads for Buildings and Other Structures ASCE 7-88, American Society of Civil Engineers 1988.

Otto, Frei. *Tensile Structures: Volume 1.* The MIT Press. 1967.

Szabo, J. and Kollar, L. *Structural Design of Cable-Suspended Roofs.* John Wiley & Sons, New York, 1984.

Vilnay, Oren. *Cable Nets and Tensegric Shells.* Ellis Horwood Limited, Israel. 1990.

APPENDIX A: SYMBOLS USED

Ax	Area of pretension cables.
Ay	Area of suspension cables.
Ec	Young's Modulus of cables.
Er	Young's Modulus of compression ring.
f	Sag in cable.
F	Axial force in cable.
fx	Sag in pretension cable.
fy	Sag in suspension cable.
Ho	Horizontal component of cable tension.
K_g	Geometric non-linear stiffness matrix.
K_l	Linear stiffness matrix.
L	Length.
Lx	Length of pretension cables.
Ly	Length of suspension cables.
nxp	Horizontal force in the pretension cable due to the pretension load.
nxq	Horizontal force in the pretension cable due to load Q.
nyp	Horizontal force in the suspension cable due to the pretension load.
nyq	Horizontal force in the suspension cable due to load Q.
P	Point load.
P_e	External load.
P_I	Internal load.
Q	Uniform load
Q_x	Portion of Q distributed to pretension cables.
Q_y	Portion of Q distributed to suspension cables.
s	Arch length of cable.
w	Vertical deflection.
w_k	Increase in vertical sag.

Δ	Change.
α	Direction of cosines.
β	Incremental direction of cosines.

APPENDIX B: DESIGN LOADS

The design loads for this project were chosen to conform with the Massachusetts State Building Code 780 CMR – Sixth Edition. They are as follows:

DEAD LOAD			
STRUCTURAL LOAD			
	ASSUMED	15.0	psf
DECK			
	20 Gage metal deck	2.5	psf
	Fiberboard insulation	1.5	psf
	Waterproof Membrane	0.7	psf
	1/2 in stone ballast	4.0	psf
	TOTAL DECK	8.7	psf
HUNG			
	ASSUMED	5.0	psf
TOTAL		28.7	psf
LIVE LOADS			
ROOF			
	Tributary load area > 600sq ft	12.0	psf
SNOW			
	Zone 2 Basic	30.0	psf
	Unbalanced 100%/50%		
WIND			
	Exposure B Zone 3		
	Suction .6 x 21 psf	12.6	psf

APPENDIX C: COMPUTER PROGRAM

The following program was written for Matlab.

```
%
%        Dan's Thesis - Tangent Stiffness Truss Method
%
% filename thesis.m
% datafiles points.dat        thesisP.dat
%           element.dat       thesisE.dat
%           area.dat
%
clear
% units are N-m

%%%%%%%%%%%%%% define variables %%%%%%%%%%%%%%%%%%%%%%%%

Hx = 100 %11030 * 15;                % Horizontal pretension force in x
direction
Hy = 100 %9437 * 12;                % Horizontal pretension force in y
direction
E = 160*10^9             % Young's Modulus
AppliedForceZ = 0 % -385267.0      % Uniform Point Load
TensionOnly = 1          % 0=NO, 1=YES

%%%%%%%%%%%%%%%%%% read data files %%%%%%%%%%%%%%%%%%%%%

%[numn x y z boundx boundy boundz] = textread('ThesisP2.dat', '%d %f %f %f %d
%d %d');
%[numm i j Area] = textread('ThesisE2.dat', '%d %d %d %f');

[numn x y z boundx boundy boundz] = textread('cableP.dat', '%d %f %f %f %d %d
%d');
[numm i j Area] = textread('cableE.dat', '%d %d %d %f');

%%%%%%%%%%%%%%%% Calculate Number of Nodes and Elements %%%%%%%%%

NumNodes = max(numn);
NumEle = max(numm);

%%%%%%%%%%%%%%% Create force and boundary arrays %%%%%%%%%%%%

P(NumNodes*3,1)=0;
B(NumNodes*3,1)=0;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%  Total loop to iterate forces %%%%%%%%%%%%%%%%%%%%%%%%%%%
%z(2)=1.1
for iter = 1:50
AppliedForceZ = AppliedForceZ -200     % incremental force
%Hx = Hx + .1                        % incremental force
%Hy = Hy + .1                        % incremental force
%Area = Area+.1                      % incremental area
%z(2) = z(2) + .1                      % incremental geometry

t=0;
K=zeros(NumNodes*3);

%%%%%%%%%%%%%%%  Create External Force Array and Boundary Conditions %%%%%%
```

```
for n=1:NumNodes
       r=(n-1)*3+1;
       P(r)=0;
       P(r+1)=0;
       P(r+2)=AppliedForceZ;
       B(r)=boundx(n);
       B(r+1)=boundy(n);
       B(r+2)=boundz(n);
end

K=zeros(NumNodes*3);

%%%%%%%%%% finding K %%%%%%%%%%%%%%%%

for n=1:NumEle

        deltax=x(j(n))-x(i(n));
        deltay=y(j(n))-y(i(n));
        deltaz=z(j(n))-z(i(n));

        length(n) = sqrt(deltax^2 + deltay^2 + deltaz^2);

       if deltax==0
              F(n)= abs(Hy*length(n)/deltay);
       elseif deltay==0
              F(n)= abs(Hx*length(n)/deltax);
       else
              F(n)= abs(Hx*length(n)/deltax+Hy*length(n)/deltay);
       end

       alpha=[deltax/length(n), deltay/length(n), deltaz/length(n)];

        k1=Area(n)*E/length(n)*(alpha'*alpha);  % linear stiffness
        k2=F(n)/length(n)*eye(3);               % geometric stiffness

       ri=(i(n)-1)*3+1;
       rj=(j(n)-1)*3+1;

        K(ri:ri+2, ri:ri+2) = K(ri:ri+2, ri:ri+2)+[k1+k2];
       K(ri:ri+2, rj:rj+2) = K(ri:ri+2, rj:rj+2)-[k1+k2];
       K(rj:rj+2, ri:ri+2) = K(rj:rj+2, ri:ri+2)-[k1+k2];
       K(rj:rj+2, rj:rj+2) = K(rj:rj+2, rj:rj+2)+[k1+k2];

end

%%%%%%%%%%%%%%%%%   REMOVING SUPPORT ROWS AND COLUMNS %%%%%%%%%%%%%%%%%%%%

for n=1:(NumNodes*3)
       m=NumNodes*3+1-n;
       if B(m)==1;
              K(:,m)=0;
              K(m,:)=0;
              K(m,m)=1;
              P(m)=0;
       end
end

U=inv(K)*P;        % calculate initital iteration displacement %

'first iterataion complete'

Pin(NumNodes * 3, 1) = 0;   %Pin is the internal forces
```

```
%%%%%%%%%%%%%%% LOOP FOR CONVERGENCE %%%%%%%%%%%%%%%%%%%

difference = U+1;
t=0;

while sum(abs(difference) >abs(U* 1e-3))>0
%for t=1:1;
       t=t+1;

       K=zeros(NumNodes*3);
       Pin(1:NumNodes*3,1)=0;

       for n=1:NumEle

              ri=(i(n)-1)*3+1;
              rj=(j(n)-1)*3+1;

              deltax2 = x(j(n)) + U(rj)   - x(i(n)) - U(ri);
              deltay2 = y(j(n)) + U(rj+1) - y(i(n)) - U(ri+1);
              deltaz2 = z(j(n)) + U(rj+2) - z(i(n)) - U(ri+2);

              length2(n) = sqrt(deltax2^2 + deltay2^2 + deltaz2^2);

              F2(n)=(length2(n)-length(n))*Area(n)*E/length(n) + F(n);

              Beta = [deltax2/length2(n), deltay2/length2(n),
deltaz2/length2(n)];

              Pint = F2(n)*Beta';

              k1 = Area(n)*E/length2(n)*(Beta'*Beta);
              k2 = F2(n)/length2(n)*eye(3);

                 if TensionOnly == 1
                        if F2(n) < 0
                           k1 = zeros(3,3);
                           k2 = zeros(3,3);
                           Pint = 0;
                           F2(n) = 0;
                        end
                 end;

                 Pin(ri:ri+2) = Pin(ri:ri+2) + Pint;
                 Pin(rj:rj+2) = Pin(rj:rj+2) -Pint;

              K(ri:ri+2, ri:ri+2) = K(ri:ri+2, ri:ri+2)+[k1]+[k2];
              K(ri:ri+2, rj:rj+2) = K(ri:ri+2, rj:rj+2)-[k1]-[k2];
              K(rj:rj+2, ri:ri+2) = K(rj:rj+2, ri:ri+2)-[k1]-[k2];
              K(rj:rj+2, rj:rj+2) = K(rj:rj+2, rj:rj+2)+[k1]+[k2];

       end

       Pei = P + Pin;

%%%%%%%%%%%%%%%%%   REMOVING SUPPORT ROWS AND COLUMNS %%%%%%%%%%%%%%%%%%%

       for n=1:(NumNodes*3)
              m=NumNodes*3+1-n;
              if B(m)==1;
                     K(:,m)=0;
                     K(m,:)=0;
                     K(m,m)=1;
```

```
                Pei(m)=0;
            end
    end

    deltaU=inv(K)*Pei;

%%%%%%%%%%%% eliminates noise from extremely small numbers %%%%%%%%%%%%

    for countit = 1:(NumNodes*3)
        if abs(U(countit)) < abs(1E-10 * max(U))
            deltaU(countit) = 0;
            U(countit) = 0;
        end
    end

    Q=U + deltaU;
    S(:,t)=U;

    S(:,t+1)=Q;
    difference = U-Q;
    U=Q;
    if t>500
        'not converging in 50 cycles'
        break
    end
end
t
AFU(:,iter)=U;              % Stores U as a function of iteration number
Ft(:,iter)=F2';             % Stores F2
U
end
```

APPENDIX E: DATA FILES

The following files were used for the cable net calculations using Matlab. The first file was named ThesisP2.dat. It contains information on the location of the joints of members and the degrees of freedom of each joint.

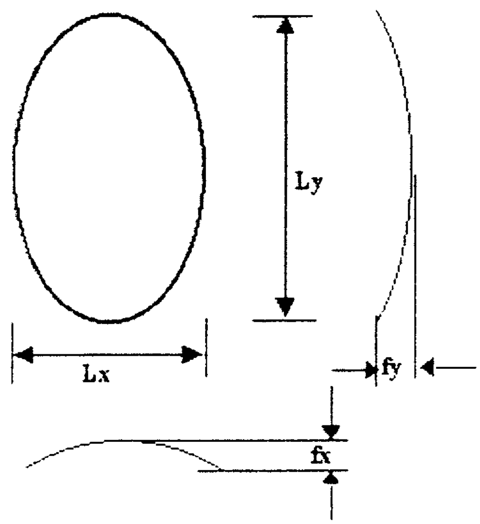
```
   Joint#    X      Y           Z      Degree of freedom X, Y, Z.
1=fixed, 0=free.
```

Joint#	X	Y	Z	DOF X	DOF Y	DOF Z
1	24	-15	-0.931901805	1	1	1
2	24	0	-2.438404877	1	1	1
3	24	15	-0.931901805	1	1	1
4	12	30	5.416411067	1	1	1
5	0	30	6.026012286	1	1	1
6	-12	30	5.416411067	1	1	1
7	-24	15	-0.931901805	1	1	1
8	-24	0	-2.438404877	1	1	1
9	-24	-15	-0.931901805	1	1	1
10	-12	-30	5.416411067	1	1	1
11	0	-30	6.026012286	1	1	1
12	12	-30	5.416411067	1	1	1
13	12	-15	0.896901852	0	0	0
14	12	0	-0.609601219	0	0	0
15	12	15	0.896901852	0	0	0
16	0	15	1.506503072	0	0	0
17	-12	15	0.896901852	0	0	0
18	-12	0	-0.609601219	0	0	0
19	-12	-15	0.896901852	0	0	0
20	0	-15	1.506503072	0	0	0
21	0	0	0	0	0	0

The following file was named ThesisE2.dat. It contains the connectivity information as well as member areas.

Joint #	I	J	Area
1	12	13	2.34E-04
2	13	14	2.34E-04
3	14	15	2.34E-04
4	15	4	2.34E-04
5	11	20	2.34E-04
6	20	21	2.34E-04
7	21	16	2.34E-04
8	16	5	2.34E-04
9	10	19	2.34E-04
10	19	18	2.34E-04
11	18	17	2.34E-04
12	17	6	2.34E-04
13	9	19	2.60E-03
14	19	20	2.60E-03
15	20	13	2.60E-03
16	13	1	2.60E-03
17	8	18	2.60E-03
18	18	21	2.60E-03
19	21	14	2.60E-03
20	14	2	2.60E-03
21	7	17	2.60E-03
22	17	16	2.60E-03
23	16	15	2.60E-03
24	15	3	2.60E-03

APPENDIX F: APPROXIMATE CALCULATIONS, RIGID EDGE RING



Approximate hand calculations for a ring cable net.

Loads:

$Qd := 14.7 \frac{lb \cdot g}{ft^2}$ $Qd = 703.84$ Pa Dead Load

$Qs := 30\ lb \cdot \frac{g}{ft^2}$ $Qs = 1.436 \cdot 10^3$ Pa Snow Load

$Qwx := -12.6 \frac{lb \cdot g}{ft^2}$ $Qwx = -603.291$ Pa Wind Load

Material Properties:

$\rho s := .283 \frac{lb}{in^3}$ Density of Steel

Dimensions

$Lx := 48$ $fx := -2.4384$ $\frac{fx}{Lx} = -0.051$ $\rho s = 7.833 \cdot 10^3 \frac{kg}{m^3}$

$Ly := 60$ $fy := 6.026$ $\frac{fy}{Ly} = 0.1$ $\frac{fx}{fy} = -0.405$

Cable Information (Cables Placed 1 Meter Apart)

$Ec := 160 \cdot 10^9$ Pa Young's Modulus for Cable

$Ax := 1.56 \cdot 10^{-5}\ m^2$ Area of Steel in X Direction Cable

$Ay := 2.17 \cdot 10^{-4}\ m^2$ Area of Steel in Y Direction Cable

$Ay \cdot \rho s = 1.7\ \frac{kg}{m}$ $\quad Ax \cdot \rho s = 0.122\ \frac{kg}{m}$

$\frac{Ay \cdot \rho s + Ax \cdot \rho s}{1\ m} = 1.822\ \frac{kg}{m^2}$ Weight of Cables

$sx := Lx \cdot\ 1 + \frac{8 \cdot fx^2}{3\ Lx^2}$ $\quad sx = 48.33$ Length of Cables Along Arc

$sy := Ly \cdot\ 1 + \frac{8 \cdot fy^2}{3\ Ly^2}$ $\quad sy = 61.614$

$Qd = 703.84\ Pa$

Assume the edge ring is infinitely rigid.

$$Qy := \frac{Qd \cdot \left[\frac{Lx^3 \cdot sx}{fx^2 \cdot (Ec \cdot Ax)} \right]}{\left[\frac{Lx^3 \cdot sx}{fx^2 \cdot (Ec \cdot Ax)} + \frac{Ly^3 \cdot sy}{fy^2 \cdot (Ec \cdot Ay)} \right]}$$

$Qy = 683.798\ Pa$ Portion of Dead Load Carried by Y direction Cables

$Qx := Qd - Qy$

$Qx = 20.042\ Pa$ Portion Carried by X Cables

$nxq := \frac{Qx \cdot Lx^2}{8 \cdot fx}$ $\quad nxq = -2.367 \cdot 10^3\ Pa$ Effective Cable Forces

$nyq := \frac{Qy \cdot Ly^2}{8 \cdot fy}$ $\quad nyq = 5.106 \cdot 10^4\ Pa$

- compression + tension

$w1 := nxq \cdot \frac{3}{16} \cdot \frac{Lx^2}{fx \cdot Ec \cdot Ax} \cdot m^3$ $\quad w1 = 0.168\ m$ Deflection due to dead load

Further Loads, results scaled since approximate calculations assume linear response.

Load Factor

Dead Load $\quad Qd = 703.84\ Pa$

$C_0 := \frac{Qd}{Qd}$ $\quad C_0 = 1$

Snow $\quad Qs = 1.436 \cdot 10^3\ Pa$

$$C_1 := \frac{Qs}{Qd} \qquad C_1 = 2.041$$

Wind 1 $\quad Qwx = -603.291\ Pa$

$$C_2 := \frac{Qwx}{Qd} \qquad C_2 = -0.857$$

Load Combinations

$C_4 := .67 \cdot C_0 + C_2 \qquad C_4 = -0.187$

$C_5 := C_0 + C_1 \qquad C_5 = 3.041$

$Pt_0 := nxq \cdot C_5 \qquad Pt_0 = -7.198 \cdot 10^3\ Pa$

$$Pt_1 := nyq \cdot C_4 \cdot \frac{-fy}{fx} \cdot \frac{Lx^2}{Ly^2} \qquad Pt_1 = -1.511 \cdot 10^4\ Pa$$

$Ptmax := max(-Pt) \qquad Ptmax = 1.511 \cdot 10^4\ Pa$

In this case, the maximum pretensioning force required to avoid zero tension in any cable is:

$nxp := Ptmax \qquad nxp = 1.511 \cdot 10^4\ Pa$ Pretension force in pretension cable

$$nyp := nxp \cdot \frac{-fx}{fy} \cdot \frac{Ly^2}{Lx^2} \qquad nyp = 9.556 \cdot 10^3\ Pa$$ Pretension force in suspension cable

Saddle Point Deformation

$$wp := \frac{nxp \cdot 3}{16} \cdot \frac{Lx^2 \cdot m^3}{fx \cdot Ec \cdot Ax} \qquad wp = -1.073\ m$$ Deflection due to pretension

$wt := wp + w1 \cdot C_5 \quad wt = -0.562\ m \qquad wt = -22.122 \circ in$ Total deflection

$nxq \cdot C_5 + nxp = 7.916 \cdot 10^3\ Pa \qquad nyq \cdot C_5 + nyp = 1.648 \cdot 10^5\ Pa$

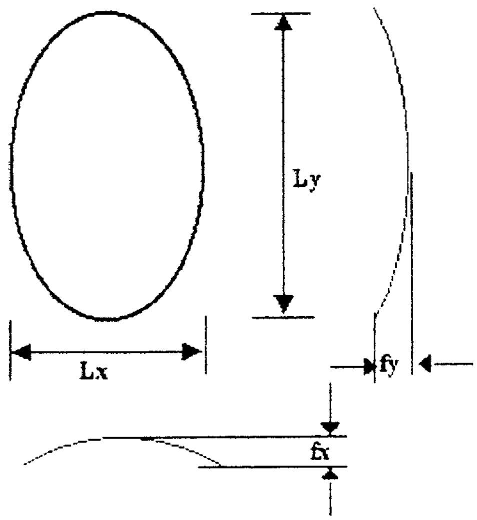
$nxq \cdot C_4 + nxp = 1.556 \cdot 10^4\ Pa \qquad nyq \cdot C_4 + nyp = -1.819 \cdot 10^{-12}\ Pa$

$$Axr := \frac{nxq \cdot C_4 + nxp \cdot 2 \cdot m}{1.5 \cdot 10^9 \frac{N}{m^2}} \qquad Ayr := \frac{nyq \cdot C_5 + nyp \cdot 2 \cdot m}{1.5 \cdot 10^9 \frac{N}{m^2}}$$

$Axr = 2.074 \cdot 10^{-5}\ m \qquad Ayr = 2.198 \cdot 10^{-4}\ m$

$Ax = 1.56 \cdot 10^{-5}\ m^2 \qquad Ay = 2.17 \cdot 10^{-4}\ m^2$

Appendix G: Approximate Calculations, Flexible Edge Ring



Loads:

$Qd := 14.7 \frac{lb \cdot g}{ft^2}$ $\quad Qd = 703.84\ Pa$ Dead Load

$Qs := 30\ lb \cdot \frac{g}{ft^2}$ $\quad Qs = 1.436 \cdot 10^3\ Pa$ Snow Load

$Qwx := -12.6 \frac{lb \cdot g}{ft^2}$ $\quad Qwx = -603.291\ Pa$ Wind Load in X Direction

Material Properties:

$\rho s := .283 \frac{lb}{in^3}$ Density of Steel

Dimensions

$Lx := 48\ m$ $\quad fx := -2.4384\ m$ $\quad \frac{fx}{Lx} = -0.051$ $\quad \rho s = 7.833 \cdot 10^3 \frac{kg}{m^3}$

$Ly := 60\ m$ $\quad fy := 6.026\ m$ $\quad \frac{fy}{Ly} = 0.1$ $\quad \frac{fx}{fy} = -0.405$

Cable Information (Cables Placed 1 Meter Apart)

$Ec := 160 \cdot 10^9\ Pa$ Young's Modulus for Cable

$Ax := 1.56 \cdot 10^{-5}\ m^2$ $\quad Ec \cdot Ax = 2.496 \cdot 10^6\ N$ Area of Steel in X Direction Cable

$Ay := 2.17 \cdot 10^{-4}\ m^2$ $\qquad Ec \cdot Ay = 3.472 \cdot 10^7\ N$ $\qquad$ Area of Steel in Y Direction Cable

$Ay \cdot \rho s = 1.7\ \frac{kg}{m} \qquad Ax \cdot \rho s = 0.122\ \frac{kg}{m}$

$\frac{Ay \cdot \rho s + Ax \cdot \rho s}{1\ m} = 1.822\ \frac{kg}{m^2}$ $\qquad$ Weight of Cables

$sx := Lx \cdot \left(1 + \frac{8 \cdot fx^2}{3\ Lx^2}\right) \qquad sx = 48.33\ m$ $\qquad$ Length of Cables Along Arc

$sy := Ly \cdot \left(1 + \frac{8 \cdot fy^2}{3\ Ly^2}\right) \qquad sy = 61.614\ m$

Ring Information

$Er := 2.75 \cdot 10^6\ Pa \qquad Ir := 1.35\ m^4$ $\qquad$ Compression Ring Modulus and Moment of Inertia

Assume the edge ring is infinitely rigid.

$$Qy := \frac{Qd \cdot \left[\frac{Lx^3 \cdot sx}{fx^2 \cdot (Ec \cdot Ax)}\right]}{\left[\frac{Lx^3 \cdot sx}{fx^2 \cdot (Ec \cdot Ax)} + \frac{Ly^3 \cdot sy}{fy^2 \cdot (Ec \cdot Ay)}\right]}$$

$Qy = 683.798\ Pa$ $\qquad$ Portion of Dead Load Carried by Y direction Cables

$Qx := Qd - Qy$

$Qx = 20.042\ Pa$ $\qquad$ Portion Carried by X Cables

$nxq := \frac{Qx \cdot Lx^2}{8 \cdot fx} \qquad nxq = -2.367 \cdot 10^3\ \frac{kg}{s^2}$ $\qquad$ Effective Cable Forces

$nyq := \frac{Qy \cdot Ly^2}{8 \cdot fy} \qquad nyq = 5.106 \cdot 10^4\ \frac{kg}{s^2}$

- compression + tension

Taking the deformation of the edge ring into consideration:

$$nbxq := \frac{nxq \cdot \frac{Ly^2}{Lx^2} - nyq}{1 + \frac{Ly^2}{Lx^2}} \qquad nbxq = -2.137 \cdot 10^4\ \frac{kg}{s^2}$$

Portion of dead load creating moment only in edge ring.

$d := \sqrt{Lx \cdot Ly}$ $\quad d = 53.666\ m$

Effective radius of similar circular plan.

$$\alpha := \frac{3 - \frac{Ly}{d}}{3 - \frac{Lx}{d}} \qquad \alpha = 0.894$$

$$nbxe := \sqrt{\alpha \cdot \frac{48 \cdot Er \cdot Ir}{d^4}} \qquad nbxe = 20.312\ Pa$$

Portion of load which creates moment only on equivalent edge ring.

$$preqx := \frac{8 \cdot \left(\frac{Lx}{-fx} - \frac{Ly}{fy} \cdot \alpha\right) \cdot \frac{1}{m}}{\left[\frac{Lx^3 \cdot sx}{fx^2 \cdot (Ec \cdot Ax)} + \frac{Ly^3 \cdot sy}{fy^2 \cdot (Ec \cdot Ay)}\right]} \qquad preqx = 232.755\ \frac{kg}{m^2 \cdot s^2}$$

Uniform force required to create ΔLx. $P = preqx/\Delta Lx$

$$nxc := \frac{-preqx \cdot Lx^2}{8 \cdot fx} \qquad nxc = 2.749 \cdot 10^4\ Pa$$

Cable forces resulting from preqx.

$$nyc := \frac{preqx \cdot Ly^2}{8 \cdot fy} \qquad nyc = 1.738 \cdot 10^4\ Pa$$

$$nbxq = -2.137 \cdot 10^4\ \frac{kg}{s^2} \qquad nbxe = 20.312\ Pa$$

$$nbxq2 := \frac{nxc \cdot \frac{Ly^2}{Lx^2} - nyc}{1 + \frac{Ly^2}{Lx^2}} \qquad nbxq2 = 9.98 \cdot 10^3\ Pa$$

$$\Delta Lx := \frac{-nbxq}{(nbxe + nbxq2)} \qquad \Delta Lx = 2.137\ m$$

Change in lenght x due to bending of the ring.

$P := preqx \cdot \Delta Lx$

Resultant force.

$P = 497.411\ Pa$

Changes in Cable Forces due to ΔL

$$Nxc2 := nxc \cdot \Delta Lx \qquad Nxc2 = 5.875 \cdot 10^4\ \frac{kg}{s^2}$$

Compression forces in the ring.

$$Nyc2 := nyc \cdot \Delta Lx \qquad Nyc2 = 3.714 \cdot 10^4\ \frac{kg}{s^2}$$

$nx := nxq + Nxc2 \qquad nx = 5.638 \cdot 10^4 \, \frac{kg}{s^2}$

Cable forces.

$ny := nyq + Nyc2 \qquad ny = 8.821 \cdot 10^4 \, \frac{kg}{s^2}$

$$n1x := \frac{nx + ny}{1 + \frac{Ly^2}{Lx^2}} \qquad n1x = 5.643 \cdot 10^4 \, \frac{kg}{s^2}$$

Cable forces causing pure compression.

$$n1y := \frac{nx + ny}{1 + \frac{Lx^2}{Ly^2}} \qquad n1y = 8.817 \cdot 10^4 \, \frac{kg}{s^2}$$

$$n2x := \frac{nx \cdot \frac{Ly^2}{Lx^2} - ny}{1 + \frac{Ly^2}{Lx^2}} \qquad n2x = -43.407 \, \frac{kg}{s^2}$$

Cable forces causing pure bending.

$n2y := -n2x \qquad n2y = 43.407 \, \frac{kg}{s^2}$

Tensile in both directions due to deformation of the edge ring.

$M1 := n2y \cdot \frac{d^2}{8} \qquad M1 = 1.563 \cdot 10^4 \, J$

Moment in ring.

$M2 := M1$

$N1 := -nx \cdot \frac{Ly}{2} \qquad N1 = -1.691 \cdot 10^6 \, N$

Axial forces in ring.

$N2 := -ny \cdot \frac{Lx}{2} \qquad N2 = -2.117 \cdot 10^6 \, N$

The Vertical Displacement of the Saddle Joint

$w0x := nxq \cdot \frac{3}{16} \cdot \frac{Lx^2 \cdot m}{fx \cdot Ec \cdot Ax} \qquad w0x = 0.168 \, m$

Deflection due to dead load.

$w1x := \frac{-3}{16} \cdot \frac{Lx}{fx} \cdot \Delta Lx \qquad w1x = 7.888 \, m$

Deflection due to edge ring deformation.

$w2x := preqx \cdot \Delta Lx \cdot \frac{3}{128} \cdot \frac{Lx^3 \cdot sx \cdot m}{fx^2 \cdot Ec \cdot Ax} \qquad w2x = 4.199 \, m$

$w := w0x + w1x + w2x \qquad w = 12.255 \, m$

Total deflection.

Further Loads, results scaled since approximate calculations assume linear response.

Load Factor

Dead Load $Qd = 703.84\ \mathrm{Pa}$

$$C_0 := \frac{Qd}{Qd} \qquad C_0 = 1$$

Snow $Qs = 1.436 \cdot 10^3\ \mathrm{Pa}$

$$C_1 := \frac{Qs}{Qd} \qquad C_1 = 2.041$$

Wind 1 $Qwx = -603.291\ \mathrm{Pa}$

$$C_2 := \frac{Qwx}{Qd} \qquad C_2 = -0.857$$

Load Combinations

$C_4 := .67 \cdot C_0 + C_2 \qquad C_4 = -0.187$

$C_5 := C_0 + C_1 \qquad C_5 = 3.041$

$$Pt_0 := nx \cdot C_5 \qquad Pt_0 = 1.714 \cdot 10^5\ \frac{kg}{s^2}$$

$$Pt_1 := ny \cdot C_4 \cdot \frac{-fy}{fx} \cdot \frac{Lx^2}{Ly^2} \qquad Pt_1 = -2.611 \cdot 10^4\ \frac{kg}{s^2}$$

$$Pt_2 := nx \cdot C_4 \qquad Pt_2 = -1.055 \cdot 10^4\ \frac{kg}{s^2}$$

$$Pt_3 := ny \cdot C_5 \cdot \frac{-fy}{fx} \cdot \frac{Lx^2}{Ly^2} \qquad Pt_3 = 4.242 \cdot 10^5\ \frac{kg}{s^2}$$

$$Ptmax := \max(-Pt) \qquad Ptmax = 2.611 \cdot 10^4\ \frac{kg}{s^2}$$

In this case, the maximum pretensioning force required to avoid zero tension in any cable is:

$$nxp_0 := Ptmax \qquad nxp_0 = 2.611 \cdot 10^4\ \frac{kg}{s^2}$$ Pretension force in pretension cable

$$nyp := nxp_0 \cdot \frac{-fx}{fy} \cdot \frac{Ly^2}{Lx^2} \qquad nyp = 1.651 \cdot 10^4\ \frac{kg}{s^2}$$ Pretension force in suspension cable

For minimum bending moment distribution under all load cases:

Factor times nd2 $$Fc := \frac{-\left[C_5 + C_4 \right]}{2} \qquad Fc = -1.427$$

$n2xp := Fc \cdot n2x \qquad n2xp = 61.935\,\frac{kg}{s^2} \qquad n2yp := -n2xp$

$$nxp_1 := \frac{\left(1+\frac{Ly^2}{Lx^2}\right)\cdot n2xp}{\frac{Ly^2}{Lx^2}\cdot\left(1+\frac{fx}{fy}\right)} \qquad nxp_1 = 170.61\,\frac{kg}{s^2}$$

$nxp := \max(nxp) \qquad nxp = 2.611\cdot 10^4\,\frac{kg}{s^2}$

$$nyp := nxp\cdot\frac{-fx}{fy}\cdot\frac{Ly^2}{Lx^2} \qquad nyp = 1.651\cdot 10^4\,\frac{kg}{s^2}$$

$$n1xp := \frac{nxp+nyp}{1+\frac{Ly^2}{Lx^2}} \qquad n1xp = 1.663\cdot 10^4\,\frac{kg}{s^2}$$

$$n1yp := \frac{nxp+nyp}{1+\frac{Lx^2}{Ly^2}} \qquad n1yp = 2.599\cdot 10^4\,\frac{kg}{s^2}$$

$n2xp = 61.935\,\frac{kg}{s^2}$

$n2yp = -61.935\,\frac{kg}{s^2}$

Force in the pretensioning (x) and suspension (y) cables under full loading with pretension.

$nx\cdot C_5 + nxp = 1.976\cdot 10^5\,\frac{kg}{s^2} \qquad ny\cdot C_5 + nyp = 2.847\cdot 10^5\,\frac{kg}{s^2}$

$nx\cdot C_4 + nxp = 1.556\cdot 10^4\,\frac{kg}{s^2} \qquad ny\cdot C_4 + nyp = 0\,\frac{kg}{s^2}$

Bending edge moments and edge normal forces:

$M1p := \frac{n2yp\cdot d^2}{8} \qquad M1p = -2.23\cdot 10^4\ J \qquad M2p := -M1p$

$N1p := -nxp\cdot\frac{Ly}{2} \qquad N1p = -7.833\cdot 10^5\ N$

$N2p := -nyp\cdot\frac{Lx}{2} \qquad N2p = -3.962\cdot 10^5\ N$

Elongations of edge axes

$$\Delta L1x := \frac{1}{48}\cdot\frac{n2yp\cdot d^4}{Er\cdot Ir\cdot\sqrt{\alpha}} \qquad \Delta L1x = -3.049\ m$$

$$\Delta L1y := \frac{3 - \frac{Ly}{\sqrt{Lx\cdot Ly}}}{3 - \frac{Lx}{\sqrt{Lx\cdot Ly}}}\cdot\Delta L1x \qquad \Delta L1y = -2.725\ m$$

Saddle Point Deformation

$$w1y := \frac{3}{16}\cdot\frac{Ly}{fy}\cdot\alpha\cdot\Delta L1x \qquad w1y = -5.088\ m$$ Deflection due to edge deformation

$$nyp = 1.651\cdot 10^4\ \frac{kg}{s^2}$$

$$w2y := nyp\cdot\frac{3}{16}\cdot\frac{Lx^2\cdot m}{fx\cdot(Ec\cdot Ax)} \qquad w2y = -1.172\ m$$

$$wp := w1y + w2y \qquad wp = -6.26\ m$$

$$wtot := wp + w \qquad wtot = 5.995\ m$$